Vol. 1 No. 2, December 2021

FUAM Journal of Pure and Applied Science

Available online at
www.fuamjpas.org.ng

An official Publication of
College of Science
Joseph Sarwuan Tarka University,
Makurdi.

FUAM Journal of Pure and Applied Science
© College of Science, Joseph Sarwuan Tarka University, Makurdi
https://fuamjpas.org.ng/
Vol.1. No. 2. December, 2021

A Probabilistic Inventory Model with Setup Cost and Decomposed Period of Holding Cost for the Optimization of Grain Storage Quantity

*[1]P.O. Agada, [2] M. Egahi and [2]D.E. Abutu

[*, 1] Department of Mathematics/Statistics/Computer Science, Joseph Sarwuan Tarka University Makurdi, Benue State, Nigeria
[2] Department of Mathematics and Computer Science, Benue State University, Makurdi, Benue State, Nigeria
*Correspondence E-mail: gadexx@yahoo.com

Received: 24/08/2021 **Accepted**: 14/10/2021 **Published online**: 17/10/2021

Abstract
The classical single period s –S policy probabilistic inventory model with setup cost, popularly known as the Newsvendor or Newsboy model has evolved over time. In this study, we addressed the problem of determining the optimal grain storage quantity for grain storage business enterprise with setup cost and decomposed period of holding stock. Modifications of the classical newsvendor model in the development of the probabilistic inventory model was adopted. Model parameters were determined from data based on expert opinion of small scale rice paddy storage traders in Makurdi, Nigeria and sensitivity analyses were performed for uniform and exponential sale distributions for comparative purpose. It was recommended that, traders with uniformly distributed sales between 1.0 and 1.5 tons (mean = 1.25 tons) would be required to store an optimal quantity of 1.0 ton of rice paddy during the period, while, those with exponential sale distribution with mean 1.25 tons, would require an optimal storage quantity of 4.0 tons. Furthermore, traders who wish to expand their business by ten times its present storage capacity of say 1.0 tons, should prepare to incur a shortage cost of N7000.00 and N116, 824.94 respectively for a uniform and exponentially distributed sale. For a storage capacity of 55 tons, they should expect a shortage cost of N115, 000.00 for a uniformly distributed sale and N57, 432.45 for an exponentially distributed sale. The study concludes that the probabilistic inventory model formulated is flexible and adaptable for any tractable distribution function of sales units and for the prediction of future shortage costs.

Keywords: Grain, inventory, model, probabilistic, storage

Introduction

The classical single period s –S policy probabilistic inventory model with setup cost, popularly known as the Newsvendor or Newsboy model has evolved over time. According to [1], extensions to this model can be classified into extension to different objectives and utility functions ([2], [3], [4], [5], [6], [7]).

Another extension is to different supplier pricing policies. These include the works of [8], [9], [10], and [11] among others. A further extension is to different news – vendor pricing policies and discounting structures among which are the works of [12], [13], [14], [15], [16], [17] Still on the list of extensions is the extension of the model to random yields. These include the works of [18], [19], [20], [21], [22], [23] and [24].

Also on the list is the extension to different states of information about demand. Some relevant researches include those of [25], [26], [27], [28] and [29] among others.

Details of further extensions such as extension to constrained multi-product, extension to multi echelon systems, extension to multi- location models, extension to models with more than one period and other extensions can also be found [1].

Some recent modifications and applications of the newsvendor model include the works of [30] on the newsvendor problem with unknown distribution, [31] on the classical newsvendor model under normal demand with large coefficients of variation, [32] on improved profit functions for newsvendor models with normally distributed demand, and the work of [33] on newsvendor problem under complete uncertainty.

This study focuses on determining the optimal grain storage quantity for a grain storage business enterprise with setup cost and decomposed period of holding stock. This necessitated a unique modification of the classical newsvendor model in the development of our probabilistic inventory model. The flexibility of the formulated model makes it adaptable for any tractable distribution function of sales units and for the prediction of future shortage costs. This justifies our reason for this study.

Methodology

In this section, we present the source and nature of data used in the research as well as the details of the model formulations.

Source and nature of data

The data for this work was sourced from experts who store and sell rice paddy on a small scale (1.0 – 1.5 tons or 10 – 10.5 bags) in Makurdi, Benue state, Nigeria. These include the current selling price of a ton (bag) of rice paddy as well as its current setup, holding and shortage cost. Details are on Table 1. Sensitivity analyses were done by varying these costs in the neighborhood of their current values for the uniform and exponential sales distribution.

Model formulation

In this section, we present the s - S Policy Model with setup cost according to [34]. This is followed by the formulation of our Probabilistic Inventory Model with setup cost and decomposed period of holding cost.

Assumptions of the s-S policy with setup inventory model

(i) Demand occurs instantaneously at the start of the period

(ii) Setup cost is incurred

Model equations of the s – S policy with setup inventory model

In this model, the total expected cost of inventory per period ($E\{\bar{C}(y)\}$) equals the sum of the setup cost (k) and the expected cost per period ($E\{C(y)\}$) as illustrated in Figure 1. That is;

$$E\{\bar{C}(y)\} = k + E\{C(y)\}$$

$$= k + h\int_0^y (y - D)f(D)dD + p\int_y^\infty (D - y)f(D)dD \qquad (1)$$

where;

k = setup cost per order, h = holding cost per held unit during the period, p = shortage or penalty cost per shortage unit during the period, f (D) = probability density function of demand, D during the period, y = order quantity and x = inventory on hand before order is placed.

This model determines the optimal order quantity (y*) that minimizes the sum of the expected holding and shortage costs. Since k is a constant, the minimum value of $E\{\bar{C}(y)\}$ must occur at y*.

It follows from (1) that;

$$p(y \leq y*) = \frac{p}{p+h} \qquad (2)$$

Observe in figure 2 that;

$$S = y* \qquad (3)$$

and $s(< S)$ is determined from the equation

$$E\{C(s)\} = E\{\bar{C}(S)\}$$

$$= k + E\{C(S)\} \qquad (4)$$

According to [34], equation (4) yields another value s_1(>S), which is discarded.

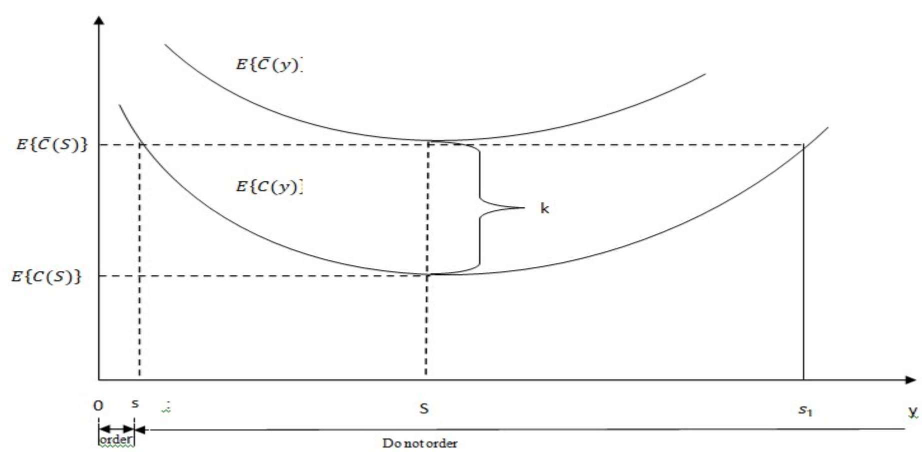

Figure 1: Optimal ordering policy for an (s – S) single period model

Assuming that x is the amount on hand before an order is placed, how much more can be ordered can be determined from the following conditions.

(i) $x < s$ (ii) $s \leq x \leq S$ (iii) $x > S$

Judging from figure 2, condition (ii) results into;

$$E\{C(x)\} \leq \min_{y>x} E\{\bar{C}(y)\} = E\{\bar{C}(S)\} \qquad (5)$$

Showing that $y* = x$. The reason being that, the cost of having x amount of stock on hand before an order is placed equals the total cost of maintaining the optimal storage quantity $(y*)$ in inventory.

Still judging from figure 2, condition (iii) also results for $y > x$, into;

$$E\{C(x)\} < E\{\bar{C}(y)\} \qquad (6)$$

Hence, $y* = x$. This means that the amount of stock (x) on hand before an order is placed should be maintained as the optimal stock $(y*)$ since it attracts a cost smaller than the total expected cost of having $y > x$ stocks in inventory.

Condition (i) is the only advantageous condition. Because x is already on hand, its equivalent cost is $\{C(x)\}$. If any additional amount $y - x$, $(y > x)$ is ordered, the corresponding cost given y, is $E\{\bar{C}(y)\}$, which includes the set up cost k. From figure 2,

$$\min_{y>x} E\{\bar{C}(y)\} = E\{\bar{C}(S)\} < E\{C(x)\} \qquad (7)$$

This means that since the total expected cost of having $y > x$ stock in inventory is less than the cost $E\{C(x)\}$, of having x stock on hand, then $y - x$ additional stock can be ordered. Hence the optimal inventory policy is to order $y - x$ amount of stock whenever x< s.

Formulation of the probabilistic inventory model with setup cost and decomposed period of holding cost

Here, we present the formulation of our probabilistic inventory model as well as it model equations. The model seeks to determine the optimal storage quantity (y*) that will minimize the total expected cost ($E\{\bar{C}(y)\}$) of keeping y units of stocks in inventory. The components of this cost are; the setup (k), holding (h) and shortage or penalty cost (p). The model was formulated by modifying the s – S policy single period inventory model with setup cost. The modifications made include the decomposition of the period of holding stocks inventory into two components. Namely, the holding cost during the fixed period of storage and the holding cost during the variable period of sales. Other modifications can be inferred from the model assumptions of both models. The formulation was done with its application to the grain storage business enterprise in mind

Model notations and parameters

Notations/parameters Notations/parameters

t_s = storage start time k = setup cost per sale

t_e = storage end time h = holding cost per head unit

t_{ss} = sales start time p = penalty or shortage cost per shortage unit

t_{se} = sales end time $f(S_a)$ = probability density function of sale, S_a during the period

t_{hs} = harvest start time y* = optimal storage units

t_{he} = harvest end time

This notations and parameters are used in understanding the dynamics of the inventory as captured in the model schematic.

Model schematic

The model schematic figure 2, shows three sub periods of the single period probabilistic inventory model. These include the storage, sales and harvest sub periods. The storage sub period is a fixed period of holding stock in inventory. The sales sub period is a variable period of holding stock in inventory while the harvest period is a sub period of no stock in inventory. The three sub periods makeup a business cycle or business period.

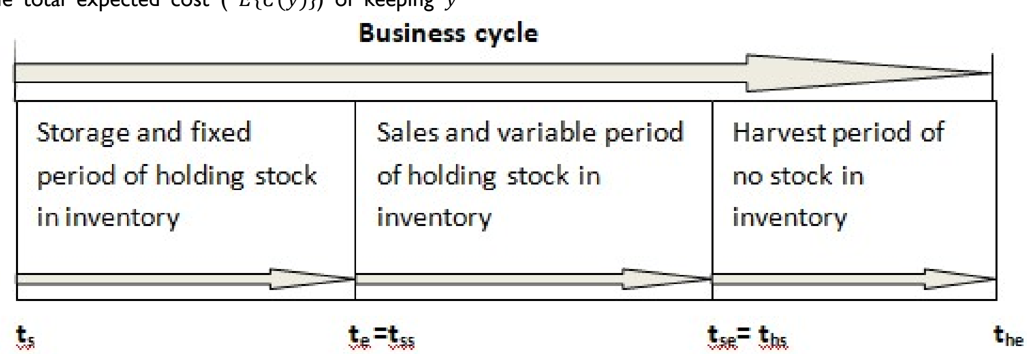

Figure 2: Model schematic of the probabilistic inventory model for grain storage business enterprise management

Model assumptions

(i) Sales begin at the end of the storage period or at the start of the sales period.

(ii) Holding cost is fixed within the storage period but changes during the sales period.

(iii) No inventory on hand before the start of the storage period.

(iv) Demand distribution of stock is same as its sales distribution

Observe in the model assumptions above, the modification made to the assumption (i) and the addition of assumptions (ii), (iii) and (iv) to the assumptions of the s – S policy inventory model. This is due to the peculiarity of our probabilistic inventory model. In this model, sales rather than demand distribution of stock is emphasized. This is because demand for grains most especially cereals, is always there. The problem is that of determining the optimal storage quantity that traders can sell (or handle) in order to minimize the sum of the setup, holding and shortage or penalty costs.

Model equations

Presented in this section are the determination of the optimal storage quantity (y*) and the total expected cost of storage, $E\{\bar{C}(y)\}$ for the uniform and exponential sale distributions.

(a) Determination of the optimal storage quantity (y*)

Since the setup cost (k) is constant and the holding cost during the storage period is fixed, we then borrow a leaf from figure 1 and write;

$$. E\{\bar{C}(y)\} = k + yh + E\{C(y)\}$$

$$= k + yh + h\int_0^y (y - S_a) f(S_a) dS_a + p\int_y^\infty (S_a - y) f(S_a) dS_a \qquad (7)$$

here all variables and parameters are as earlier defined.

We seek to determine the optimal storage quantity (y*) that minimizes the total expected cost, $E\{\bar{C}(y)\}$ per period. Hence differentiating equation 7 with respect to y, equating to zero and simplifying results into;

$$p(y \le y*) = \frac{p-h}{p+h}, \; p \ge h \qquad (8)$$

where,

$$p(y \le y*) = \int_0^{y^*} f(S_a) dS_a \; , \int_{y*}^\infty f(S_a) dS_a = 1 - \int_0^{y^*} f(S_a) dS_a \qquad (9)$$

This imply that the chance of having a storage quantity (y) between 0 and the optimal quantity (y*) in inventory exist if the penalty cost (p) is greater than or equal to the holding cost (h).

For a uniform (a,b) sales distribution where a = minimum sales unit and b = maximum sales unit;

$$p(S_a \le y*) = \int_a^{y*} \frac{1}{b-a} dS_a$$

$$= \frac{y* - a}{b-a} \qquad (10)$$

Equating equations (8) and (10) we have;

$\frac{p-h}{p+h} = \frac{y* - a}{b-a}$ and

$$y* = a + \frac{p-h}{p+h}(b-a) \qquad (11)$$

For an exponential sales distribution with $mean = \frac{1}{\lambda}$,

$$f(S_a) = \lambda e^{-\lambda S_a}, \quad S_a > 0 \qquad (12)$$

$$p(S_a \le y*) = F(y*) = 1 - e^{-\lambda y*}, \; S_a > 0 \qquad (13)$$

Equating (8) with (13) we have;

$\frac{p-h}{p+h} = 1 - e^{-\lambda y*}$ and

$$y* = -E[S_a] * \ln\left(1 - \frac{p-h}{p+h}\right) \qquad (14)$$

Observe from figure 1 that if we hold S units of inventory at the start of the period (when the inventory is empty) then, $y = S$.

Observe also that;

$$\min_{y \ge S} E\{\bar{C}(y)\} = E\{\bar{C}(S)\} < E\{\bar{C}(y)\}_{y>S} \qquad (15)$$

This shows that the total cost of storing S units in inventory at the start of the period, is the least cost one would ever incur amidst all costs of storing $y \ge S$ units in inventory.

Our inventory policy therefore is to;

Store $y* = S$ units in inventory at the start of the period.

As with the s – S policy model, the optimality of our model is guaranteed because of the convex nature of the associated cost function.

(b) Determination of the total expected cost of storage, $E\{\bar{C}(y)\}$

Recall from equation (7) that;

$$E\{\bar{C}(y)\} = k + yh + h\int_0^y (y - S_a) f(S_a) dS_a + p\int_y^\infty (S_a - y) f(S_a) dS_a$$
$$= k + yh + p(S_a \le y) yh - hE[S_a] + p - pE[S_a] + p(S_a \le y) yp$$

$$= k + p + yh + p(S_a \le y) y(p+h) - E[S_a](p+h) \qquad (16)$$

(i) For the aforementioned uniform (a,b) sales distribution, recall that

$$p(S_a \le y*) = \frac{y* - a}{b-a}$$

and

$$E[S_a] = \frac{a+b}{2}$$

It follows from equation (15) that;

$$E\{\bar{C}(y*)\} = k + p + y*(h) + (p+h)(\tfrac{y*(y-a)}{b-a} - \tfrac{(a+b)}{2}) \quad (17)$$

(ii) For the aforementioned exponential sales distribution, recall from equation (13) that

$p(S_a \leq y*) = 1 - e^{-\lambda y*}$, mean = $1/_{\lambda}$, $S_a > 0$

$$E\{\bar{C}(y*)\} = k + p + y*(h) + (p+h)(y*(1-e^{-\lambda *}) - 1/_{\lambda}) \quad (18)$$

Predicting shortage cost (p) per unit when y* optimal units of storage are held in inventory

For any distribution of sales, infer from equation (8) that

$$p(0 \leq y*) = \frac{p-h}{p+h},\ p \geq h$$

and

$$p = \frac{h(1+p(0\leq y*))}{1-p(0\leq y*)} \quad (19)$$

It follows that the shortage or penalty cost (p) for the uniform and exponential sales distribution are respectively;

$$p = \frac{h(2a-y*-b)}{y*-b} \quad (20)$$

and

$$p = \frac{h(2-e^{-\lambda y*})}{e^{-\lambda y*}} = h(2e^{\lambda y*} - 1) \quad (21)$$

Sensitivity analysis

We perform sensitivity analyses using equations (16) and (17) for the uniform and exponential sales distribution. This is done in order to see the response of the total expected cost of storage, $E\{\bar{C}(y)\}$ of holding y units of storage in inventory to changes in the holding cost (k) and the shortage or penalty cost (p). The former is used to address the holding cost concern of traders or farmers who store their grains in rented storage houses while the latter is used to address the concern of traders or farmers who would like to see the effect of various shortage or penalty costs (p) on the total expected cost of inventory. Further in the sensitivity analyses, equation (19) is used in predicting the shortage or penalty cost (p) given the cumulative probability of holding y* units of optimal inventory for the uniform and exponentially distributed sales. This is to enable traders or farmers to know beforehand, what they should expect of their shortage cost given their distribution of sales.

Result and Discussion

Following the information sourced from the opinion of small scale traders of rice paddy involved in the storage business enterprise, model results were obtained for assumed uniform and exponential distribution of sales. These two distributions as earlier mentioned, were chosen to reflect traders or farmers with equal sale chances (random sales) and those whose sale chances decreases with increase in storage quantity exponentially. As shown in table 1, a small scale rice paddy storage business trader in Makurdi metropolis, with uniformly distributed sales between 1.0 and 1.5 tons (mean = 1.25 tons) and holding, shortage and setup cost of N5000.00, N240, 000.00 and N1000.00 per ton would be required to store an optimal quantity of 1.0 ton of rice paddy during the period for a minimum total expected cost of storage. For comparative purpose, a rice paddy storage business man of equivalent capacity (mean = 1.25 tons) with exponential sales distribution would have exponential distribution parameter $\lambda = 0.8$. Maintaining the inventory model parameter estimates as in the uniform sales distribution, this trader would be required to maintain an optimal storage quantity of 4.0 tons during the period for a minimum total expected cost. See table 1 below for details.

Table 1: Optimal storage unit and model parameter estimates for the uniform and exponential sales distribution

Sales distribution and parameters	Optimal storage units (tons)	Holding cost per unit (N), per period	Shortage cost per unit (N)	Setup cost per unit (N)
Uniform (a,b),a = 1.0, b = 1.5	1.00	5000.00	240000.00	1000.00
Exponential (lamda), lamda = 0.8	4.00	5000.00	240000.00	1000.00

Sensitivity analyses on the response of the total expected cost of inventory to changes in the holding and shortage costs were done for the uniform and exponential sales distributions. See tables 2 and 3. Table 2 shows that as the holding cost changes between N0.00 and N10, 000.00 per ton, the total expected cost of inventory ranged between N236,163.65 and N245,861.06 for the uniform sales distribution and between N864,432.85 and N930,267.70 for the exponential sales distribution. Observe the high total expected cost of inventory that would be incurred by traders with the exponential sales distribution. This is because increase in holding cost is as a result of more storage being held in inventory. This favours exponentially distributed sales as the chance of sales decreases with increase in storage quantity. The same explanation goes for the high shortage costs incurred by traders with exponentially distributed sales as shown in table 3.

Table 2: Distribution of uniform and exponential total expected costs for varying holding costs

Holding cost per unit (N)	Total expected cost (N) for uniform sales	Total expected cost (N) for exponential sales
0.00	236163.65	864432.85
2500.00	238588.00	880891.56
5000.00	241012.35	897350.28
7500.00	243436.70	913808.99
10000.00	245861.06	930267.70

Table 3: Distribution of uniform and exponential total expected costs for varying shortage costs

Shortage cost (N)	Total expected cost (N) for uniform sales	Total expected cost (N) for exponential sales
100000.00	103826.90	395430.15
120000.00	123424.83	467133.02
140000.00	143022.75	538835.90
160000.00	162620.67	610538.77
180000.00	182218.59	682241.65
200000.00	201816.51	753944.53
220000.00	221414.43	825647.40
240000.00	241012.35	897350.28

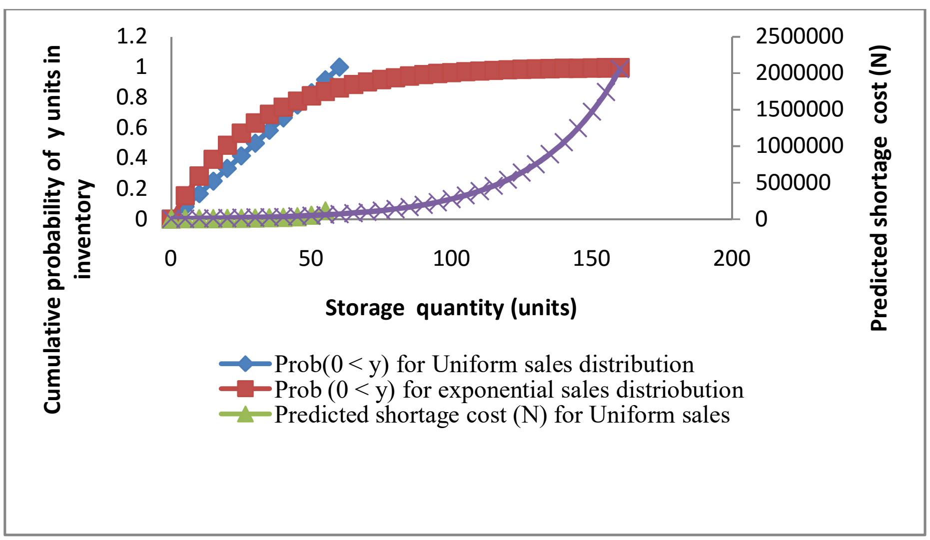

Figure 3: Graphical relationship between predicted shortage cost and cumulative probability of storage quantity

Further sensitivity analyses were performed to predict the shortage cost that would be incurred by traders with uniform and exponential sales for varying optimal storage units. Table 4 depicts a distribution of predicted shortage costs for uniformly distributed sales between 0 and 60 tons with the mean of 30 tons. It also depicts a distribution of predicted shortage costs of exponentially distributed sales with mean 30 tons for comparative purpose. Observe from this table that the cumulative probability of having between 0 to y* units of storage in inventory increases with increase in storage units and so does the predicted cost of shortage.

Table 4: Distribution of predicted shortage cost for uniform and exponential sales across varying storage units

Uniformly distributed sale between 0 and 60 tons with mean 30 tons.			**Exponentially distributed sales with mean 30 tons**		
Optimal storage (sales) units (y*)	Prob(0 < y) for Uniform sales distribution	Predicted shortage cost (N) for Uniform sales	Optimal storage (sales) units (y)	Prob (0 < y) for exponential sales distriobution	Predicted shortage cost (N) for exponential sales
0	0.00	5000.00	0	0.00	5000.00
5	0.08	5909.09	5	0.15	6811.64
10	0.17	7000.00	10	0.28	8951.47
15	0.25	8333.33	15	0.39	11478.97
20	0.33	10000.00	20	0.49	14464.36
25	0.42	12142.86	25	0.57	17990.59
30	0.50	15000.00	30	0.63	22155.65
35	0.58	19000.00	35	0.69	27075.26
40	0.67	25000.00	40	0.74	32886.13
45	0.75	35000.00	45	0.78	39749.72
50	0.83	55000.00	50	0.81	47856.73
55	0.92	115000.00	55	0.84	57432.45
60	1.00	-	60	0.86	68742.93
-	-	-	65	0.89	82102.46
-	-	-	70	0.90	97882.25
-	-	-	75	0.92	116520.76
-	-	-	80	0.93	138535.89
-	-	-	85	0.94	164539.36
-	-	-	90	0.95	195253.71
-	-	-	95	0.96	231532.38
-	-	-	100	0.96	274383.42
-	-	-	105	0.97	324997.50
-	-	-	110	0.97	384781.02
-	-	-	115	0.98	455395.13
-	-	-	120	0.98	538801.94
-	-	-	125	0.98	637319.02
-	-	-	130	0.99	753683.80
-	-	-	135	0.99	891129.64
-	-	-	140	0.99	1053475.66
-	-	-	145	0.99	1245232.85
-	-	-	150	0.99	1471729.45
-	-	-	155	0.99	1739258.98
-	-	-	160	1.00	2055255.11

The graphical relationship between the predicted shortage cost and the cumulative probability of optimal storage quantity is shown in figure 3. Observe also that if our small scale rice paddy trader wishes to expand his business by ten times its present storage capacity of say 1.0 tons, then from table 4, he would incur a shortage cost of N7000.00 and N116, 824.94 for a uniform and exponentially distributed sale respectively. For a capacity of 55 tons, he should expect a shortage cost of N115, 000.00 for a uniformly distributed sales and N57, 432.45 for an exponentially distributed sale. Equation (18) reveals the flexibility of the model, as it makes it adaptable for predicting future shortage costs for any distribution function of sales units.

Conclusion

The study concludes that a probabilistic inventory model with setup cost and decomposed period of holding cost has being successfully developed in the study for the optimization of grain storage quantity. The flexibility of the model makes it adaptable for any tractable distribution function of sales units and for the prediction of future shortage costs.

The study recommends that small scale rice paddy storage business traders in Makurdi metropolis with uniformly distributed sales between 1.0 and 1.5 tons (mean = 1.25 tons) would be required to store an optimal quantity of 1.0 ton of rice paddy during the period while, those with exponential sales distribution with mean 1.25 tons would require an optimal storage quantity of 4.0 tons. In addition, if theses rice paddy traders wish to expand their businesses by ten times its present storage capacity of say 1.0 ton, then they should prepare to incur a shortage cost of N7000.00 and N116824.94 respectively for a uniform and exponentially distributed sale. For a capacity of 55 tons, they should expect a shortage cost of N115, 000.00 for a uniformly distributed sale and N57, 432.45 for an exponentially distributed sale.

For further research, our probabilistic inventory model with setup cost, decomposed period of holding stocks in inventory and triangular sales distribution should be considered. Moreover, a Monte Carlo Simulation Approach to handling sparse sales or demand data should also be considered for our model.

Declaration of conflicting interests

The authors declared no potential conflicts of interest

References

[1] Khouja, M. 1999. The single-period (news-vendor) problem: literature review and suggestions for future research. Omega, *International Journal of Management Science*. 27, 537-535.

[2] Kabak I, Schi, A. 1978. Inventory models and management objectives. Sloan Management Reveiew. 10:53-59.

[3] Finley D.R, Liao W.M. 1981. A general decision model for cost, volume and profit analysis under uncertainty: a comment. *Account Review* 56, 400-403.

[4] Ismail B, Louderback J. 1979. Optimizing and satisfying in stochastic cost volume and profit Analysis. *Decision Science*. 10, 205-217.

[5] Atkinson, A.A. 1979. Incentives, uncertainty, and risk in the newsboy problem. Decision Science. 10, 341-357.

[6] Eeckhoudt L, Gollier C, Schlesinger H. 1995. The risk averse (and prudent) newsboy. *Management Science*. 41, 786-794.

[7] Jucker J.V, Rosenblatt M.J. 1985. Single-period inventory models with demand uncertainty and quantity discounts: behavioral implications and a new solution procedure. *Naval Research Logistics.* 32, 537-550.

[8] Pantumsinchai P, Knowles T,W. 1999. Standard container size discounts and the single-period Inventory problem. *Decision Science*. 22, 612 - 619.

[9] Lin C, Kroll D.E. 1997. The single-item newsboy problem with dual performance measures and quantity discounts. *European Journal of Operations Research*. 100, 562-565.

[10] Kabak I, Weinberg C. 1972.The generalized newsboy problem, contract negotiations and secondary vendors. *Institute of Industrial Engineering Transactions* 4, 154-157.

[11] Whitin T.M. 1955. Inventory control and price theory. *Management Science*. 2, 61 - 68.

[12] Lau A, Lau H. 1988. Maximizing the probability of achieving a target profit level in a two-product newsboy problem. *Decision Science*. 19, 392 - 408.

[13] Polatoglu LH. 1991. Optimal order quantity and pricing decisions in single-period inventory systems. *International Journal of Production Economics*. 23, 175-185.

[14] Khouja M. 1996. A Note on the newsboy problem with an emergency supply option. *Journal of Operations Research Society*. 47, 1530-1534.

[15] Khouja M. 1995. The newsboy problem under progressive multiple discounts. *European Journal of Operations Research*. 84, 458-466.

[16] Karlin S. 1958. One stage inventory models with uncertainty. In: Arrow K, Karlin S, Scarf H), editors. Studies in the mathematical theory of inventory and production. Stanford, CA: Stanford University Press, 1958.

[17] Noori A.H, Keller G. 1986. One-period order quantity strategy with uncertain match between the

amount received and quantity requisitioned. *Institute for Operations Research and Management.* 24, 1-11.

[18] Ehrhardt R, Taube L. 1987. An inventory model with random replenishment quantity. *International Journal of Production Research*. 25, 1795 - 803.

[19] Gerchak Y, Vickson R.G, Parlar M. 1988. Periodic review production models with variable yield and uncertain demand. *Institute of Industrial Engineering Transactions*. 20, 144 - 150.

[20] Henig M, Gerchak Y. (1990). The structure of periodic review policies in the presence of random yield. *Journal of Operations Research Society*. 38, 634-643.

[21] Parlar M, Wang D. 1993. Diversification under yield randomness in inventory models. European Journal of *Operations Research*. 66, 52 - 64.

[22] Jain K, Silver E.A. 1995. The single period procurement problem where dedicated supplier capacity can be reserved. *Naval Research Logistics*. 42, 915 - 934

[23] Benzion, U., Cohen, Y and Shavit, T. 2010. Newsvendor problem with unknown distribution. *Journal of Operations Research Society* 61, 1022-1031.

[24] Gallego G, Moon I. 1993. The distribution free newsboy problem: review and extensions. *Journal of Operations Research Society*. 44, 825-834.

[25] Moon I, Choi S. 1995. The distribution free newsboy problem with balking. *Journal of Operations Research Society*. 46, 537-542.

[26] Reyniers D. 1991). A high-low search for a newsboy problem with delayed information feedback. *Journal of Operations Research Society*. 38, 838 - 846.

[27] Petrovic D, Petrovic R, Vujosevic M. 1996. Fuzzy models for the newsboy problem. *International Journal of Production Economics*. 45, 435 - 441.

[28] Gerchak Y, Mossman D. 1992. On the effect of demand randomness on inventories and costs. *Operations Research*. 40, 804- 807.

[29] Hill R. 1997. Applying Bayesian methodology with a uniform prior to the single period inventory model. *European Journal of Operations Research*. 98, 555-562.

[30] Khouja, M and Mehrez, A. 1996. A multi-product constrained newsboy problem with progressive multiple discounts. *Computers and Industrial Engineering*. 30, 95 - 101.

[31] Halkos, E.G. and Kevork , I.S. 2012. The classical newsvendor model under normal demand with large coefficient of variations. Munich Personal RePEc Archive Paper No 40414 http://mpra.ub-muenchen.de/40414/

[32] Hu, J. and Munson, C.J. 2011. Improved profit functions for newsvendor models with normally distributed demand. *International Journal of Procurement Management* 4, 20-36.

[33] Gaspars-Wieloch,H. 2017. Newsvendor problem under complete uncertainty: *Operations Research*. 25(3), 561-585.

[34] Taha, H. A. 2011. Operations Research an Introduction. Prentice Hall, New York. Pp 612-623.

Cite this article

Agada P.O., Egahi M. and Abutu D.E. (2021). A Probabilistic Inventory Model with Setup Cost and Decomposed Period of Holding Cost for the Optimization of Grain Storage Quantity. FUAM Journal of Pure and Applied Science,**1**(**2**):49-57