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Numerical Simulation of the Nonlinear Difusion-Type Equation in a Two Phase Media with Functional Porosity and Permeability Models

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Abstract

This work presents a mathematical model for a radial flow of a two phase fluid in a porous media (petroleum reservoir). Because the porosity and permeability were pressure dependent, the resulting diffusivity equation was a nonlinear pressure diffusion type equation. The rock porosity and permeability are mathematically modelled as functions of pressure. The resulting nonlinear pressure diffusion type equation is solved numerically retaining its nonlinearity with an explicit backward-forward finite difference method using MATLAB software. A constant porosity and permeability model was first simulated to test the code and to serve as a baseline as it has well known analytic results. The different models for the porosity and permeability were simulated and results obtained and discussed. The results showed that the pressure distribution in a reservoir is dependent on the porosity and permeability models. The slopes from the pressure distribution curves were used to calculate the Darcy flux towards the wellbore. The inverse pressure porosity and permeability model was observed to be the model with the highest fluid yield towards the wellbore as it recorded the least pressure at the well bore at all times, that is, 1000 Pascal at t=10 hours, 500 Pascal at T=25 hours, 400 Pascal at T=50 hours and 250 Pascal at t=100 hours. It had a Darcy flux at T=20 hours, r=1000 as 13.33m/hr and at r=6000 meters as 9.11m/hr. Also at T=50 hours, r=1000 meters as 6.33m/hr and r=6000 meters as 6.94m/hr.

Keywords: permeability, porosity, nonlinear pressure diffusion, two phase media, numerical simulation

Introduction

A porous medium is a solid or collection of solid bodies with sufficient open space (pores or voids) in or around the solids to enable a fluid to pass through or around them [1]. The skeletal portion of the material is called the "matrix" or "frame". A fluid can only flow through a porous medium only if at least some of the pores are inter-connected. The inter-connected pore space is termed the effective pore space, while the whole of the pore space is termed the total pore space. Example of a porous medium is an aquifer from which ground water is pumped, reservoirs which yield oil and/or gas [2]. The flow of fluids through a porous media is therefore the manner in which the fluid molecules move from one point of the media to another through the pores (voids) [3]. Flow and transport phenomenon in porous media arise in fields of science and engineering, ranging from agricultural, biomedical, construction, ceramic, chemical and petroleum engineering to food and soil sciences, and powder technology [4].

The problem of fluid flow in porous media can be better understood if one takes a look at fluid flows in pipes and water channels. It is rather easy to take measurements such as the length and diameter of a pipe and compute its flow capacity and pressure as a function of distance and time. In pipes the volume of fluid delivered at the open end depends on the pressure difference and partly on the cross-sectional area of the pipes if the pressure is high [5]. The flow of water in channels also depends on pressure difference but is often coded in terms of topography of the land. When the pressure in a pipe is very high, the pipe may either burst or the rate of fluid delivery will increase. Under the ground there are no such pipes, no clear-cut flow paths that can be used for measurements. The fine capillary like pores through which the fluid flows are not of any uniform cross-section nor can they be said to be rigid. The consequence is that the flow parameters such as porosity of the media through which fluids flow beneath the earth surface and the permeability of the fluid may in fact depend on the fluid pressure [6]. The models of fluid flow incorporating functional dependence of media and fluid properties on pressure results to complex and nonlinearity of the governing equations.

Due to complex nature of porous media (for example, a reservoir) various researchers were attracted to tackle the problems related to this topic and formulated different relations for studying diffusion of fluids in porous media. However, most of the researchers derived their models on the assumption of a constant diffusion coefficient (diffusivity constant) [7]. The diffusivity in practice may not be constant and may depend on temperature, concentration, pressure, amongst others. In some attempts, where nonlinearity is retained in the models, they are indirectly avoided at the solution stage through the process of linearization [8]. This is like abandoning the problem at hand to pursue the ghost of it. Also, due to the complex nature of multiphase flow, nonlinearity of their governing equations and reservoir intricacies, finding analytical solutions to practical fluid flow problems is impossible. Therefore, the only means by which such models can be solved is by using numerical methods such as finite difference or finite element.

In this work the nonlinearity of media and fluids are fully retained making the usual diffusion equation to be a nonlinear diffusion-type of equation and the numerical approach used in the solution also fully retain the nonlinearity.

Materials and Methods

Governing laws and equations

One of the basic laws governing fluid flow through a porous media is the mass conservation law,

$$-\frac{\partial(\rho q r)}{\partial r} = r\frac{\partial(\rho\emptyset)}{\partial t}. \quad (1)$$

where ρ = density of the fluid,

q = Darcy's velocity

$\emptyset$ = porosity

r = radial distance

t = time taken

For isothermal fluid flow the mass conservation law can be written as;

$$r\,\frac{\partial(\rho\emptyset)}{\partial t} = r\left[\rho\emptyset(c_\emptyset + c_f)\,\frac{\partial p}{\partial t}\right]. \quad (2)$$

where,

$$c_\emptyset = \frac{1}{\emptyset}\left(\frac{\partial\emptyset}{\partial p}\right)_t. \quad (3)$$

$c_\emptyset$ = isothermal compressibility of porosity

while, $c_f = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_t$. (4)

c_f = isothermal compressibility of fluid

$$c_\emptyset + c_f = c_t\,. \quad (5)$$

c_t = total compressibility

Another basic law governing fluid flow is the Darcy Equation,

$$q = -\frac{k}{\mu}\,\frac{dp}{dr}. \quad (6)$$

where p = pressure of fluid

k = rock permeability

μ = dynamic fluid viscosity

Substituting equation (5) and (6) into (2) results to;

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}\,\rho r\frac{\partial p}{\partial r}\right] = \rho\emptyset c_t\frac{\partial p}{\partial t}\,. \quad (7)$$

This is the governing equation for transient radial flow of a fluid through a porous rock [9].

Consider a two phase flow where the fluids are immiscible and there is no mass transfer between the phases. One phase (for example, oil) wets the porous medium more than the other (for example, gas) and is called the wetting phase and indicated by a subscripts, o. The other phase is termed the non-wetting phase and is indicated by g. In general, water is the wetting fluid relative to oil and gas, while oil is the wetting phase relative to gas. Several new quantities peculiar to multiphase flow, such as saturation, capillary pressure and relative permeability must be introduced. The saturation of a fluid phase is defined as the fraction of the void volume of a porous medium filled by the phase [10,11]. The fact that the two fluids jointly fill the voids implies the relation

$$s_o + s_g = 1. \quad (8)$$

where s_0 and s_g are the saturations of the wetting and non-wetting phases respectively. Also due to the curvature and surface tension of the interface between the two phases, the pressure difference is given by the capillary pressures;

$$p_c = p_g + p_o\,. \quad (9)$$

Empirically, the capillary pressure is a function of saturation s_0 . Except for the accumulation term, the same derivation that led to equation (6) applies to the mass conservation equation for each fluid phase. Taking into account that there is no mass transfer between phases in the immiscible flow, mass is conserved within each phase and each phase has its own density, ρ and Darcy's velocity, q. The mass conservation equation for each phase is

$$-\frac{\partial(\rho q_o r)}{\partial r} = r\frac{\partial(\emptyset\rho_o s_o)}{\partial t}. \quad (10)$$

$$-\frac{\partial(\rho_g q_g r)}{\partial r} = r\,\frac{\partial(\emptyset\rho_g s_g)}{\partial t}. \quad (11)$$

Darcy's law can be generalized for the two phase flow by also including a relative permeability factor (k_r) for each phase and each phase having its own viscosity and pressure [12,13].

$$q_o = \frac{-kk_{ro}}{\mu_o}, q_g = \frac{-kk_{rg}}{\mu_g}\,\frac{\partial p_g}{\partial r}. \quad (12)$$

The two phase flow equation thus reads;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{ro}}{\mu_o}\rho_o r\frac{\partial p_o}{\partial r}\right) = \emptyset\rho_o s_o c_t\frac{\partial p_o}{\partial t}$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{rg}}{\mu_g}\,\rho_g r\frac{\partial p_g}{\partial r}\right) = \emptyset\rho_g s_g c_t\frac{\partial \rho_g}{\partial t}. \quad (13)$$

In the oil and gas reservoirs, the capillary pressure is always much less than p_o or p_g and so we can say $p_o \approx p_g$ and $p_c = 0$. Our equation reduces to a single equation. We will use the equation for the oil phase and replace p_o with p. Equation 21 reduces to;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kk_{ro}}{\mu_o}\rho_o r\frac{\partial p}{\partial r}\right) = \emptyset\rho_o s_o c_t\frac{\partial p}{\partial t}. \quad (14)$$

If the fluid properties are kept constant throughout the flow regime such that only the rock properties (porosity and permeability) varies, ignoring the subscripts equation (14) can be simplified as follows;

Bringing out the constants and using product rule to open the brackets,

$$\frac{\rho k_{ro}}{r\mu}\frac{\partial}{\partial r}\left(kr\frac{\partial p}{\partial r}\right) = \emptyset\rho sc_t\frac{\partial p}{\partial t}. \quad (15)$$

$$\frac{k_{ro}}{r\mu}\left[r\frac{\partial k}{\partial r}\frac{\partial p}{\partial r} + k\frac{\partial p}{\partial r} + kr\frac{\partial^2 p}{\partial r^2}\right] = \emptyset sc_t\frac{\partial p}{\partial t}. \quad (16)$$

From the first term on the right in the bracket,

$$r\frac{\partial k}{\partial r}\frac{\partial p}{\partial r} = r\frac{\partial k}{\partial p}\frac{\partial p}{\partial r}\frac{\partial p}{\partial r}$$

$$= r\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2. \quad (17)$$

Substituting equation (20) into (19) and simplifying further,

$$\frac{k_{ro}}{\mu}\left[\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2 + \frac{k}{r}\frac{\partial p}{\partial r} + k\frac{\partial^2 p}{\partial r^2}\right] = \emptyset sc_t\frac{\partial p}{\partial t}. \quad (18)$$

$$\frac{\partial p}{\partial t} = \frac{k_r}{s\mu c_t}\frac{k}{\emptyset}\left[\frac{1}{k}\frac{\partial k}{\partial p}\left(\frac{\partial p}{\partial r}\right)^2 + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial r^2}\right]. \quad (19)$$

Equation (22) will be the mathematical model of the two phase diffusion through a porous medium. It is a nonlinear differential equation since the porosity and permeability depends on pressure. That is, $\emptyset = \emptyset(p)$ and $k = k(p)$. Equation (22) is too complex to solve analytically, therefore, it will be solved using the Finite Difference Method (FDM).

Model discretization

Discretization using the backward forward difference scheme [14], [15]: The backward-forward scheme is an explicit finite difference scheme, giving a first order convergence in time and second order convergence in space [16]. The spatial interval [0, R] and time interval [0, T] are partitioned into respective finite grid as follows:

$$r_i = (i-1)\Delta r, \qquad i = 1,2 \dots I+1, \ where\ \Delta r = \frac{R}{I} \quad . \quad (20)$$

$$t_n = (n-1)\Delta t, \qquad i = 1,2 \dots N+1,$$

$$\text{where } \Delta t = \frac{T}{N}. \quad (21)$$

The numerical solution is an approximation to the exact solution that is obtained using a discrete representation to the Partial Differential Equation (PDE) at the grid point r_i on the discrete spatial mesh at every time level t_n.The numerical solution at a grid point $p(r_i, t_n)$ is denoted by the symbol p_i^n and the derivatives are replaced by suitable difference quotients(to get an algebraic equation which can be solved).The backward-forward analog is;

Forward difference

$$\Delta p_i^n = p_{i+1}^n - p_i^n. \quad (22)$$

Backward difference

$$\nabla p_i^n = p_i^n - p_{i-1}^n. \quad (23)$$

Central difference

$$\delta^2 p_i^n = p_{i+1}^n - 2p_i^n + p_{i-1}^n = \Delta(\nabla p_i^n). \quad (24)$$

$$\Delta(\nabla) = \nabla(\Delta)$$

Thus, substituting eq.25-27 into 22 results to

$$\frac{p_i^{n+1} - p_i^n}{\Delta t} = \frac{\alpha k(p_i^n)}{\emptyset(p_i^n)}\left[\frac{1}{k(p_i^n)}\frac{\partial k(p_i^n)}{\partial p}\frac{\Delta p_i^n}{\Delta r}\frac{\nabla p_i^n}{\Delta r} + \frac{\Delta p_i^n + \nabla p_i^n}{2\Delta r} + \frac{\delta^2 p}{(\Delta r)^2}\right] \quad . \quad (25)$$

Along the left boundary

$$\Delta p_1^n = \frac{p_1^n - p_{1-1}^n}{\Delta t} = 0. \quad (26)$$

And along the right boundary

$$\nabla p_I^n = \frac{p_{I+1}^n - P_I^n}{\Delta r} = 0. \quad (27)$$

These conditions together with (28) results to

$$p_i^{n+1} = p_i^n + \frac{\alpha\Delta t}{(\Delta r)^2}\frac{k(p_i^n)}{\emptyset(p_i^n)}\left[\frac{1}{k(p_i^n)}\frac{\partial k(p_i^n)}{\partial p}(\Delta p_i^n)(\nabla p_i^n) + \frac{\Delta r}{2r}(\Delta p_i^n + \nabla p_i^n) + \delta^2 p\right]. \quad (28)$$

$p_i^1 = f(r)$, for this simulation,

$$p_i^1 = 4000\cos(\pi r_i). \quad (29)$$

For i=1,

$$p_1^{n+1} = p_1^n - H\frac{k(p_1^n)}{\emptyset(p_1^n)}\left((p_2^n - p_1^n) - \frac{p_2^n - p_1^n}{2}\right) \quad (30)$$

For i=2 to I,

$$p_i^{n+1} = p_i^n + H\frac{k(p_i^n}{\emptyset(p_i^n)}[\frac{1}{k(p_i^n)}\frac{\partial k(p_i^n)}{\partial p}(\Delta p_i^n)(\nabla p_i^n) + \frac{1}{2(i)}(\Delta p_i^n + \nabla p_i^n) + \delta^2 p] . \quad (31)$$

For i=I+1,

$$p_{I+1}^{n+1} = p_{I+1}^n + H\frac{k(p_{I+1}^n)}{\emptyset(p_{I+1}^n)}[\frac{1}{2(I+1)} - 1](p_{I+1}^n - p_I^n \quad (32)$$

where,

$$H = \frac{\alpha\Delta t}{(\Delta r)^2} . \quad \text{and} \quad \frac{\Delta r}{r} = \frac{1}{i} . \quad (33)$$

The numerical solution is obtained by calculating p_i^{n+1} recursively for n = 1,. . .,N by the use of (29), (30), (31) and (32). In the usual diffusion equation, where k and $\emptyset$ are constants, equation (30), (31), and (32) are completely ready for simulation. In this work, these functions are functional of pressure. To make any further head way one must make models of the functional dependence on pressure. This is the major departure from the known diffusion equation. Another departure from the known diffusion equation is the presence of the square on the derivative of pressure with respect to time.

Porosity and Permeability Models

Porosity and permeability are two of the primary properties that control the movement and storage of fluids in rocks [17, 18]. They both depend on pores in the porous medium [19]. Change of the pressure inside the pore spaces during production can affect its porosity and permeability [20].

Constant porosity and constant permeability: The constant porosity, $\emptyset$ and permeability, k, model is giving as;

$$\emptyset = 0.18 \, , k = 1.0\text{e} - 10. \quad (34)$$

Inverse pressure porosity and permeability model: The inverse pressure porosity, $\emptyset$ and permeability, k model is giving as;

$$\emptyset = \emptyset_{re}\frac{p_{re}}{p}, k = \frac{k_{re}p_{re}}{p} . \quad (35)$$

Sinusoidal porosity and constant permeability model: The sinusoidal porosity, $\emptyset$ and constant permeability, k model is defined as;

$$\emptyset = \frac{\emptyset_{re}\sin(p)}{\sin(p_{re})} ,k = 1.0e^{-10}. \quad (36)$$

Linear porosity and constant permeability Model: The linear porosity, $\emptyset$ and constant permeability, k model is giving as;

$$\emptyset = \emptyset_{re}\frac{p}{p_{re}},k = 1.0e^{-10}. \quad (37)$$

Darcy flux towards the wellbore

Equation (6) can be rearranged and integrated as;

$$\int_{p_w}^{p_r}\partial p = \frac{q\mu}{kk_r}\int_{r_w}^{r}\partial r \quad (38)$$

$$p_r - p_w = \frac{q\mu}{kk_r}(r - r_w) \quad (39)$$

where p_r = pressure at a distance r,

p_w = wellbore pressure,

r = radial distance from wellbore,

r_w = wellbore radius,

k_r =relative permeability.

$$p_r = \frac{q\mu}{kk_r}r + p_w. \quad (40)$$

A plot of pressure against radial distance from the wellbore will have an intercept on the pressure axis equal to the wellbore pressure p_w, q is the Darcy fluid velocity towards the wellbore at the radial distance, r, and a pressure gradient V.

$$V = \frac{q\mu}{kk_r} \quad (41)$$

From the knowledge of V, the Darcy flux is computed as;

$$q = \frac{kk_rV}{\mu} \quad (42)$$

Simulation Parameters

The simulation parameters and the conversion factor to SI units are given in Table 1.

Table 1: Values for Rock and Fluid Properties.

Parameter	Value	Unit
Viscosity, μ	0.001	Pas
Reference porosity, $\emptyset_{re}$	0.5	_
Reference permeability, k_{re}	$1.0e^{-10}$	m^3
Reference pressure, ρ_{re}	4000	Pa
Total compressibility, c	$5.07e^{-14}$	Pa^{-1}
Relative permeability, k_r	0.1	_
Oil saturation, s	0.5	_
Length of reservoir, R	10000	M

Simulation procedure

The simulation equations (Eq. 29-32) were coded in a MATLAB programing language. The various models of the porosity, $\emptyset$ and permeability, k are contained in separate scripts and are combined to give the required model. The input parameters contained in Table I were initialized with their corresponding values in the program. For each model chosen the simulation procedure is as follows;

1. start program: discretize the diffusion type equation(Eq.22-25)
2. input the parameters needed to described the reservoir (Table 1)
3. set the initial conditions (Eq. 29)
4. set the boundary conditions (Eq. 30 and 32)
5. calculate porosity and permeability from previous step (Eq. 34-37)
6. calculate the pressure at each time step from the result obtained in the previous step using equation (31)
7. display results
8. plot graphs
9. end program.

Figure I illustrates the computational cycle for each functional script.

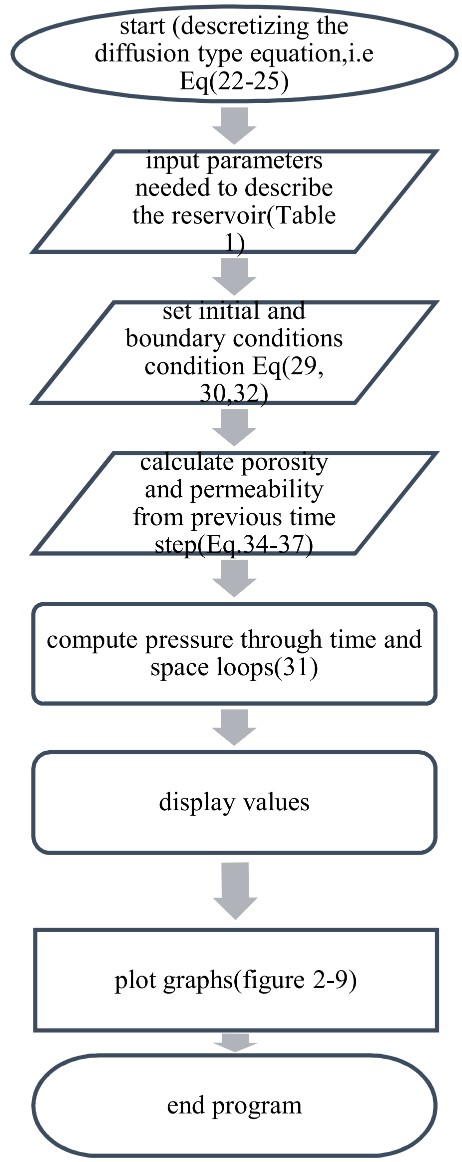

Figure 1: Flow Chart Showing the Simulation Procedure.

Results and Discussion

Constant porosity and permeability model

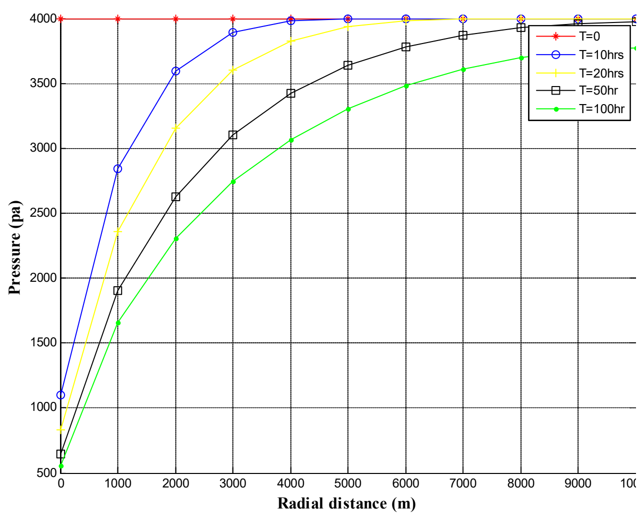

Figure 2: Pressure Distribution for the Constant Porosity and Permeability Model.

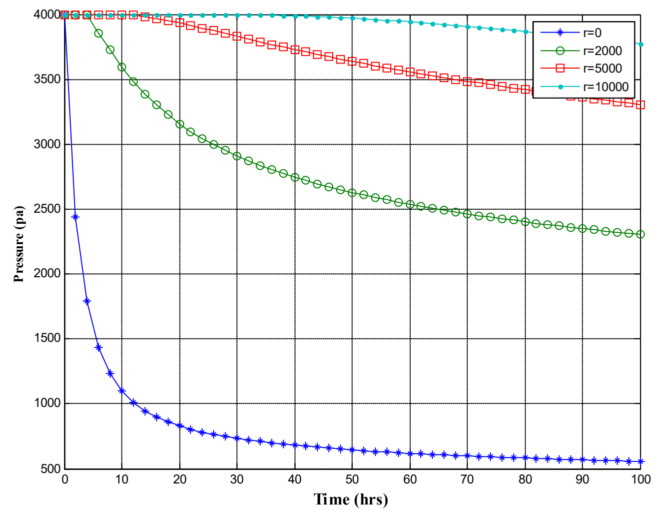

Figure 3: Pressure Profile for the Constant Porosity and Permeability Model.

When the porosity, $\emptyset$ and permeability, k are constant the simulation equation (22) reduces to a linear regular diffusion type equation (17). The concentration of the diffusing agent should have a Gaussian distribution while the Gaussian picks keep reducing in time [21]. Our simulation equation only appears like a diffusion type equation since pressure is not a diffusing agent rather an agent that causes flow of fluids. If a fluid is to flow towards the origin (wellbore) the pressure distribution should follow the Gaussian curves for $r < 0$. That is, the pressure distribution curves should be a mirror reflection of concentration diffusion curves [22]. Our simulation results reproduce this situation very well as shown in figure 2. For $t_i > t_j$, we expect the subsequent pressure distribution curves to lie below those of t_j. This is correctly obtained in the simulation (Figure 2). Similarly, for fixed locations, we expect the pressure profile to follow a Gaussian decrement with time as shown in Figure 3. The constant porosity and permeability model is a baseline model with known analytic result. The correct reproduction of the simulation results gives us the confidence that while the porosity and permeability are no more constants, the simulation results may contain useful physics.

Inverse Porosity and Pressure Permeability Model

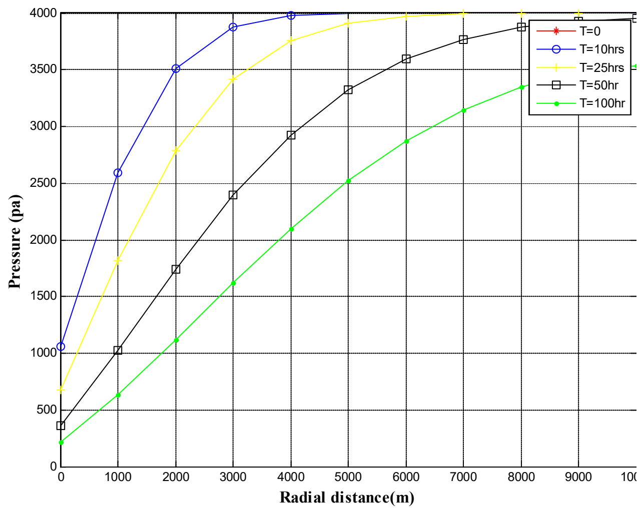

Figure 4: Pressure Distribution for the Inverse Pressure Porosity and Permeability Model

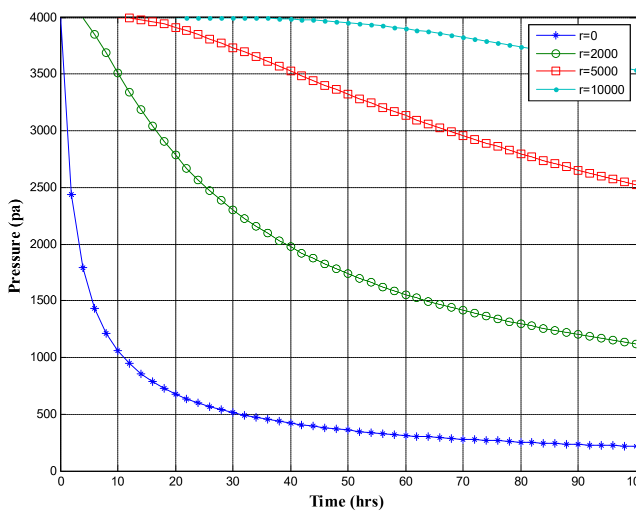

Figure 5: Pressure Profile for the Inverse Pressure Porosity and Permeability Model

Figure 4 presents the pressure distribution in the reservoir for the inverse porosity and permeability model. An inverse porosity and permeability model means that the reservoir has its porosity and permeability decreasing with increasing distances from the well bore. As such the pressure drop across the reservoir should be less than that obtained from the constant porosity and permeability model as observed in Figure 4. The curves from the pressure profile of Figure 5, shows a faster decrease in the reservoir pressure with time on comparison with figure 3. This can be seen as the pressure dropped below 500 Pascal at 100 hours while in Figure 3 it was still above 500 Pascal. Same behavior is observed for the other locations. Since the pressure of the reservoir is decreasing with time the porosity and permeability increased with time resulting in more fluid yield at the well.

Sinusoidal Porosity and Constant Permeability Model

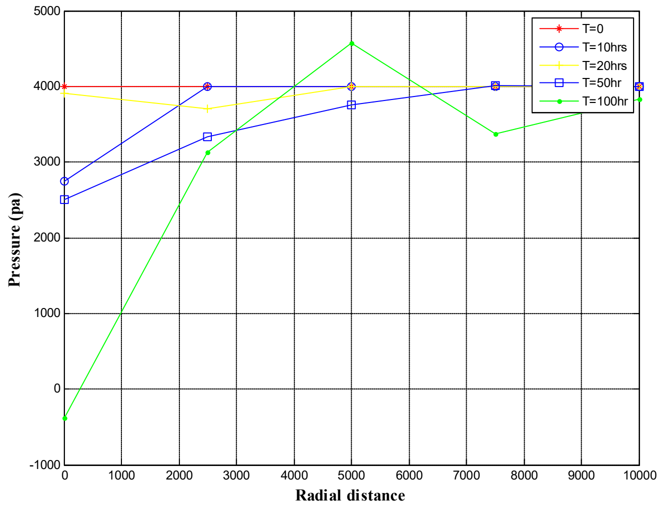

Figure 6: Pressure Distribution for the Sinusoidal Porosity and Constant Permeability Model.

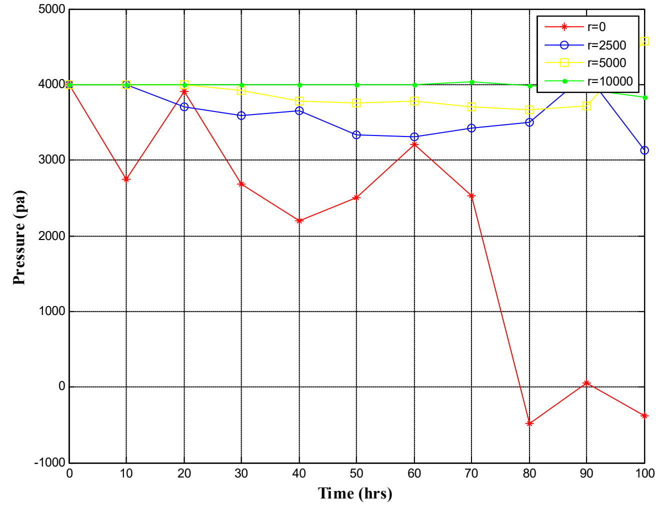

Figure 7: Pressure Profile for the Sinusoidal Porosity and Permeability Model.

The sinusoidal porosity and constant permeability model gives a pressure distribution curve (Figure 6) that is sinusoidal as the pressure transient travels through the reservoir to the boundary. Also its pressure profile at a fixed distance (Figure 7) decreased sinusoidal with time. The pressure curves show that the pressure gradient occurred faster and is greater than that obtained from the constant porosity and permeability model.

Linear Porosity and Constant Permeability Model

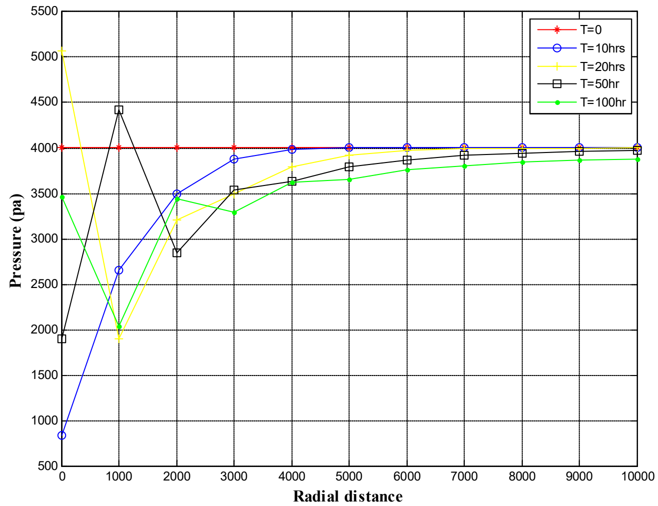

Figure 8: Pressure Distribution for the Linear Porosity and Constant Permeability Model.

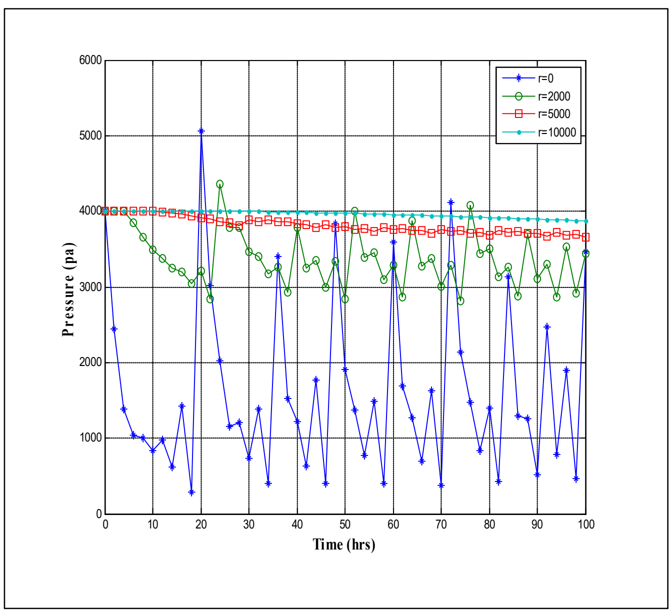

Figure 9: Pressure Profile for the Linear Porosity and Constant Permeability Model.

The pressure curves obtained using the linear porosity and constant permeability models (Figure 8 and Figure 9) shows that the reservoir experienced a few hours of steady drop in pressure followed by an irregular, haphazard behavior in the pressure drop. Because the porosity of the reservoir is increasing with pressure, locations farther away from the well will experience pressure drop faster than those before them. More fluid will flow out of distant locations than closer locations. That is, the porosity towards the well will decrease with time leading to pressure build up towards the well bore. This situation can occur when debris flow and block the pores during production. The pressure profile also shows that after a few hours of production there was an irregular, haphazard decrease in pressure with time at each location. The pressure at the well bore built up above the rest of the reservoir at $t = 50$ hours and $t = 100$ hours , showing that at such times, fluid flow to the reservoir will cease. The pressure profile is observed to be irregular due to pressure buildup and drop as the fluid flows in the reservoir.

Darcy flux towards the well bore

Table 2 and table 3 shows the Darcy flux obtained a for 20 hours and 50 hours respectively at r=1000 meters and r=6000 meters 0f each model. The Darcy flux determines the fluid flow direction [23]. Three types of pressure gradient can be observed from table 2 and table 3, negative, zero and positive pressure gradients. Models with negative pressure gradient gave rise to negative Darcy flux which means fluid flow is in a direction opposite the well bore. Models with positive pressure gradient resulted in positive Darcy flux which means fluid flow is in the direction of the wellbore. Models with zero pressure gradient resulted to zero Darcy which means no fluid motion in a particular direction.

Table 2: The slope and the Darcy velocity at 20 hours for two locations

Model equation number	Slope at r=1000	Slope at r=6000	q_{2000} ($\times 10^{-9}$m/hr)	q_{5000} ($\times 10^{-9}$m/hr)
(34)	1.037	0.938	10.37	9.38
(35)	2.250	1.036	22.50	10.36
(36)	-0.264	-	-2.64	-
(47)	1.903	0.524	19.03	5.24

Table 3: The slope and the Darcy velocity at 50 hours for two locations

Model equation Number	Slope at r=1000	Slope at r=6000	Q_{2000} ($\times 10^{-9}$m/hr)	q_{5000} ($\times 10^{-9}$m/hr)
37	1.536	0.458	15.36	4.58
38	-0.977	-3.923	-9.77	-39.23
39	1.382	-0.568	13.82	-5.68
40	2.033	0.526	20.33	5.26

Conclusion

The nonlinear pressure diffusion–type equation with pressure dependent porosity and permeability models was derived and simulated using an explicit backward-forward finite difference scheme providing numerical solutions with the aid of MATLAB. The numerical results predicted the pressure distribution in a petroleum reservoir under the different porosity and permeability models. For a wellbore to be producing, the porosity and permeability should yield pressure curves that are increasing with increasing distances from the well and decrease with increasing time. Porosity and permeability models that resulted in increasing well pressure caused pressure buildup at the wellbore and resulted to little or no fluid flow to the well. Fluid flows to the wellbore when the Darcy flux towards it is positive. This research can be applied in the oil and gas industry to predict pressure behavior in reservoirs and make investment decisions, production and maintenance decision.

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Declaration of conflicting interests

The authors declared no potential conflicts of interest

References

[1] Jin, Y. and Chen, K. 2019.**Fundamental Equations for Primary Fluid Recovery from Porous Media.** *Journal of fluid Mechanics*. 860:300-317.

[2] Ehirim, O. E and Ngustav, S.C. 2019.**Mathematical Modelling and Volumetric Estimation of a Three Phase Fluid Flow in a Porous Media Using Eclipse 100. A Case Study of a Niger Delta Reservoir.** *International Journal of Chemistry and Chemical Processes.* 5(1): 1-8.

[3] Iftikhar, A., Nadeen, M., and Bilal, C. 2014. **Fractional Diffusion Model for Transport through Porous Media. 5th International Conference on Porous Media and their Applications in Science, Engineering and Industry.** *ECI Symposium Series*. (3)

[4] Selcuk, E., Fowler, S.J. *et al*. 2017.**An analytical Model of Porosity-Permeability for Porous and Fractured Media.** *Transport in Porous Media*. 120(2): 327-358.

[5] Longcang Shu 2020.**Numerical Simulation of Groundwater. Hohai University Lecture Notes** https://www.iahr.org. Accessed 20th December, 2022.

[6] Scheidegger, A.E. 2020.**The Physics of Flow Through Porous Media.** 3rd Edition, Toronto, University of Toronto Press. Pp 53, .34-58.

[7] Shen, L., Cui, T., Liu, H., Zhu, Z., Zhang, H., Chen, Z., Yang, B., He, R. and Liu, H. **Numerical Simulation of Two-Phase Flow in Naturally Fractured Reservoirs Using Dual Porosity Method on Parallel Computers.** Proceedings of the International Conference on High Performance Computing in Asia Pacific Region, Guangzhou, China. 14th January 2019, 91-100.

[8] Cheng,Y. 2017.**Reservoir Simulation COMSOL Multiphysics Users Guide.** *Petroleum Engineering-Upstream*. Pp 15, 10-15.

[9] Gasmi, S. and Nouri, F.Z. 2015. **Numerical Simulation for Two Phase Flow in a Porous Medium.** *Boundary Value Problem*. 2015,7.

[10] Ahmed, T. and Nathan, M. 2011.**Advanced reservoir Management and Engineering.** 2nd edition

Houston, Gulf Professional Publishing. Pp 720, 712-715.

[11] Jose M.C., Stefano P. *et al.* 2014.**Numerical Simulation of Two Phase Fluid Flow.** *Journal of Petroleum Exploration and Production Technology*. 4(3): 233-243.

[12] Nmebu, C.G, 2014. **Modelling the Pressure Distribution in a Reservoir Undergoing MEOR for a 2-Dimensional Flow System**. *International journal of Emerging Technology and Advanced Engineering*. 4(6): 402-411.

[13] Opoku, A., Ohenewaa, K.D. and Eric, T.B. 2019.**A Comparative Study Between Finite Difference Explicit and Implicit Method for Predicting Pressure Distribution in a Petroleum Reservoir.** *International Journal of Engineering science and Technology*.11(4):23-40.

[14] Chen, Z., Huan, G., and Ma, Y. 2006.**Computational Methods for Multiphase Flows in Porous Media. Philadelphia, PA, USA.** Computational Science and Engineering Series, 2, Pp 1-282.

[15] Jain, M.K., Iyengar, S.R.K. and Jain, R.K. 2016.**Computational Methods for Partial Differential Equations.** 2nd Edition Newyork, New Age International Publishers, Pp 240, 5-260.

[16] Winrich, Lonny Bee 1968.**An Explicit Method for Numerical Solution of a Nonlinear Diffusion Equation Retrospective Thesis and Dissertifications,3269.**https://lib.dr.iastate.edu/rtd/3269. Accessed 28th December, 2022.

[17] Chunguang, L., Guanhua, S., Mingwei, G. and Shuilin, W. 2019. **Permeability of Porous Media Under Confining Pressure.** *Geophysics Journal International*. 216(3): 2017-2024.

[18] Goral, J., Panja, P., Deo, M., Andrew, M., Linden, S., Scharz, T. and Wiedman, A. 2020. **Confinement Effect on Porosity and Permeability of Shales.** *Science* Report. 10, 49.

[19] Liu, J. and Lui, X. 2001.**The Effect of Effective Pressure on Porosity and Permeability of Low Permeability Porous Media.** *Journal of Geomechanics*. 7(1):41-44.

[20] Bohnsack, D., Potten, M., Freitag, S., Einsiedl, F. and Kai Z. 2021. **Stress Sensitivity of Porous and Permeability Under Varying Hydrostatic Stress Conditions for Different Carbonate Rock type of the Geothermal Malm Reservoir in Southern Germany.** *Geothermal Energy Journal.* 9(15): 1-59.

[21] Schmelter, S. and Bar, M. 2020. **Numerical Simulation Validation and Analysis of Two Phase Slug Flow in Large Horizontal Pipes.** *Journal of Flow Measurement and Instrumentation*. 73, (101722): 1-5.

[22] Abdullah, M.A., Panda, A., Gupta, S., Joshi, S., Singh, A. and Rao, N. 2019. **Multi-Scale Method for Modeling and Simulation of Two Phase Flow in Reservoir Using MRST.** *Petroleum and Coal*. 61(3): 546 – 558.

[23] Ahmed, T. and McKinney, P.D. 2005. **Advanced Reservoir Engineering.** 1st edition, Oxford, Gulf Professional Publishing. Pp 363, 1-363.

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