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# Exponential Type Estimators of Finite Population Mean in Stratified Random Sampling

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**Abstract**

In this paper, alternative hybrid exponential regression-cum-ratio type estimators of finite population mean in stratified random sampling utilizing two auxiliary variables are proposed. The proposed estimators are an extension of estimator to stratified random sampling. The expressions for the bias and Mean Square Error (MSE) of the estimators are derived and asymptotic properties of the proposed estimators investigated. A comprehensive simulation study to show the efficacy of the estimators as compared to conventional estimators was carried out using Coefficient of Variation as a performance measure. The results of the simulation study have shown that the proposed estimators were not only asymptotic and more efficient, but they also produced estimates that were more precise than most of the existing estimators considered in this study.

**Keywords:** Variables, Bias, Coefficient of Variation, Hybrid Estimators, Mean Square Error

## Introduction

When auxiliary information are utilized at the estimation stage, the ratio, product and regression methods are usually employed for a given sampling design [2]. Under simple random sampling, several authors have suggested estimators and, of course, introduced certain modifications to estimators of population characteristics especially when information on auxiliary variable that is highly correlated with the variable of interest is known. These include works by [3], [4], [5], [6], [7], etc.

Since stratified random sampling has been proved for providing more efficient estimates of population parameters over SRS, several authors have extended application of modified estimators in SRS to stratified random sampling. For instance, [8] suggested a ratio-cum- product estimator of mean using correlation coefficient between study and auxiliary variates in stratified sampling whereas, [9] proposed a ratio-cum-product estimator in stratified random sampling. Using information on the coefficient of kurtosis of auxiliary variables, [10] proposed an improved dual to ratio cum dual to product estimator of finite population mean in stratified sampling,[11] estimator in stratified sampling is modification of[10] estimator using two transformed auxiliary variables.

In fact,[5] estimator of finite population in Stratified sampling is a turning point in the use of auxiliary variable for increasing the precision of estimators of population characteristics when the correlation between auxiliary variable and the study variable is weak. Based on this [12]suggested hybrid



type exponential ratio-cum product estimator of population mean in stratified sampling.

The stratified sampling scheme is preferable in situations where the population is heterogeneous in nature but it is possible to create non-overlapping groups with similar characteristics (homogeneous units). This study constructs alternative hybrid exponential regression-cum-ratio type estimators of population mean in stratified random sampling by considering the correlation coefficient as the parameter space.

## Materials and Methods

### *Preliminaries and notations*

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of size $N$ and it is partitioned into L strata of size,$N_h(h = 1, 2, \dots, L)$. Let $Y$ be the study variable of interest and $X$ and $Z$ be two supplementary variables taking values $y_{hi}, x_{hi}$ and $z_{hi}(h = 1, 2, \dots, L; i = 1, 2, \dots N_h)$ on $i^{th}$ unit of the $h^{th}$ stratum. A sample of size $n_h$ is drawn at random without replacement from each stratum which comprises a sample of size,$n = \sum_{h=1}^{L} n_h$. Basically, $N_h$: Population size of $h^{th}$stratum, $n_h$: Sample size of $h^{th}$ stratum, $\bar{Y}_h = \frac{1}{N_h}\sum_{h=1}^{N_h} y_{hi}$ : $h^{th}$ stratum population mean of the study variable Y, $\bar{X}_h = \frac{1}{N_h}\sum_{h=1}^{N_h} x_{hi}$ : $h^{th}$ stratum population mean of the auxiliary variable X, $\bar{Z}_h = \frac{1}{N_h}\sum_{h=1}^{N_h} z_{hi}$ : $h^{th}$ stratum population mean of the auxiliary variable Z, $\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$: population mean of study variable Y, Y, $\bar{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}x_{hi}$: population mean of auxiliary variable X, $\bar{Z}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}z_{hi}$: population mean of the auxiliary variable Z, $\bar{y}_h=\frac{1}{n_h}\sum_{h=1}^{n_h}y_{hi}$: sample mean for the study variable Y for $h^{th}$ stratum, $\bar{x}_h=\frac{1}{n_h}\sum_{h=1}^{n_h}x_{hi}$: sample mean for auxiliary variable X for $h^{th}$ stratum, $\bar{z}_h=\frac{1}{n_h}\sum_{h=1}^{n_h}z_{hi}$: sample mean for auxiliary variable Z for $h^{th}$ stratum, $W_h=\frac{N_h}{N}$: stratum weight of $h^{th}$ stratum, $\bar{y}_{st}=\sum_{h=1}^{L}W_h\bar{y}_h$: unbiased estimator of population mean of study variable Y, $\bar{x}_{st}=\sum_{h=1}^{L}W_h\bar{x}_h$: unbiased estimator of population mean of auxiliary variable X, $\bar{z}_{st}=\sum_{h=1}^{L}W_h\bar{z}_h$: unbiased estimator of population mean of auxiliary variable Z, $S_{y,h}^2=\frac{1}{N_h-1}\sum_{h=1}^{L}(y_{hi}-\bar{Y}_h)^2$: $h^{th}$ stratum population variance of study variable Y, $S_{x,h}^2=\frac{1}{N_h-1}\sum_{h=1}^{L}(x_{hi}-\bar{X}_h)^2$: $h^{th}$ stratum population variance of auxiliary variable X, $S_{z,h}^2=\frac{1}{N_h-1}\sum_{h=1}^{L}(z_{hi}-\bar{Z}_h)^2$: $h^{th}$ stratum population variance of auxiliary variable Z, $S_{y,x,h}=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(y_{hi}-\bar{Y}_h)(x_{hi}-\bar{X}_h)$: $h^{th}$ stratum population covariance between Y and X, $S_{y,z,h}=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(y_{hi}-\bar{Y}_h)(z_{hi}-\bar{Z}_h)$: $h^{th}$ stratum population covariance between Y and Z, $S_{x,z,h}=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(x_{hi}-\bar{X}_h)(z_{hi}-\bar{Z}_h)$: $h^{th}$ stratum population covariance between X and Z, $\bar{x}_h^*=(1-g_h e_{x,h})\bar{X}_h$: $h^{th}$ stratum sample mean of transformed auxiliary variable X, $\bar{z}_h^*=(1-g_h e_{z,h})\bar{Z}_h$: $h^{th}$ stratum sample mean of transformed auxiliary variable Z. We also define: $\boldsymbol{\rho_{HHH}}$: The parameter subspace, $(0.7<\rho_1,\rho_2,\rho_3<1)$; $\boldsymbol{\rho_{HLL}}$ :the parameter subspace, $(0<\rho_2,\rho_3<0.5\cup 0.7<\rho_1<1)$; $\boldsymbol{\rho_{LLL}}$: The parameter subspace, $(0<\rho_1,\rho_2,\rho_3<0.5)$ and $\boldsymbol{\rho_{\bar{L}LL}}$ : the parameter subspace, $(-0.5<\rho_1,\rho_2,\rho_3<0)$, where, $\boldsymbol{\rho_1}$:Correlation coefficient between Y and Z, $\boldsymbol{\rho_2}$: Correlation coefficient between X and Z, $\boldsymbol{\rho_3}$: Correlation coefficient between Y and X.

***Existing estimators in literature***

***Ratio-cum-product estimator in stratified sampling***

[9] proposed a ratio-cum-product estimator in stratified random sampling. It is presented as follows:

$$\bar{y}_{RP.st}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \tag{1}$$

$$Bias(\bar{y}_{RP.st})=\bar{Y}_{st}\sum_{h=1}^{L}\theta_h W_h^2\left(C_{xh}^2-C_{yxh}-C_{xzh}+C_{yzh}\right) \tag{2}$$

$$MSE(\bar{y}_{RP.st})=\bar{Y}_{st}^2\sum_{h=1}^{L}\theta_h W_h^2\left[C_{y,h}^2+C_{x,h}^2-2\left(C_{y,x,h}-C_{y,z,h}+C_{x,z,h}\right)\right] \tag{3}$$

***Improved ratio-cum-product estimator***

Using information on the coefficient of kurtosis of auxiliary variables, [10] proposed an improved estimator of finite population mean in stratified sampling. It is given by:

$$\bar{y}_{Tet.st}=\bar{y}_{st}\left[\frac{\sum_{h=1}^{L}W_h\{\bar{X}_h+\beta_{2h}(x)\}}{\sum_{h=1}^{L}W_h\{\bar{x}_h+\beta_{2h}(x)\}}\right]\left[\frac{\sum_{h=1}^{L}W_h\{\bar{z}_h+\beta_{2h}(z)\}}{\sum_{h=1}^{L}W_h\{\bar{Z}_h+\beta_{2h}(z)\}}\right] \tag{4}$$

$$Bias(\bar{y}_{Tet.st})=\bar{Y}_{st}\sum_{h=1}^{L}\theta_h W_h^2\left[\left(\frac{S_{zh}^2}{\bar{X}_{1h}^2}-\frac{S_{xzh}}{\bar{X}_{1h}\bar{Z}_{1h}}\right)+\frac{1}{\bar{Y}}\left(\frac{S_{yzh}}{\bar{Z}_{1h}}-\frac{S_{yh}}{\bar{X}_{1h}}\right)\right] \tag{5}$$

$$MSE(\bar{y}_{\mathrm{Tet}.st})=\sum_{h=1}^{L}\theta_h W_h^2\left[S_{y,h}^2+R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1S_{yzh}-2R_1R_2S_{xz}\right] \tag{6}$$

where,

$$R_1=\frac{\bar{Y}}{\bar{X}};\ R_2=\frac{\bar{Y}}{\bar{Z}};\ \bar{X}_{1h}=\frac{\bar{Y}}{\sum_{h=1}^{L}W_h\{\bar{X}_h+\beta_{2h}(x)\}};\ \bar{Z}_{1h}=\frac{\bar{Y}}{\sum_{h=1}^{L}W_h\{\bar{Z}_h+\beta_{2h}(z)\}}$$

***Dual to ratio-cum-product estimator***

Estimator [12] in stratified sampling is modification of [10] estimator using two transformed auxiliary variables. It is given as:

$$\bar{y}_{YT.st} = \bar{y}_{st}\left[\frac{\sum_{h=1}^{L} W_h\{\bar{x}_h^* + \beta_{2h}(x)\}}{\sum_{h=1}^{L} W_h\{\bar{X}_h + \beta_{2h}(x)\}}\right]\left[\frac{\sum_{h=1}^{L} W_h\{\bar{z}_h^* + \beta_{2h}(z)\}}{\sum_{h=1}^{L} W_h\{\bar{Z}_h + \beta_{2h}(z)\}}\right] \quad (7)$$

$$Bias(\bar{y}_{YT.st}) = \bar{Y}_{st}\sum_{h=1}^{L}\theta_h W_h^2 g_h\left[\frac{g_h}{\bar{Z}_{1h}}\left(\frac{S_{zh}^2}{\bar{Z}_{1h}} - \frac{S_{xzh}}{\bar{X}_{1h}}\right) + \frac{1}{\bar{Y}}\left(\frac{S_{yzh}}{\bar{Z}_{1h}} - \frac{S_{yh}}{\bar{X}_{1h}}\right)\right] \quad (8)$$

$$MSE(\bar{y}_{YT.st}) = \sum_{h=1}^{L}\theta_h W_h^2\left[S_{y,h}^2 + g_h^2 R_{12}^2 S_{xh}^2 + g_h^2 R_{13}^2 S_{zh}^2 - 2g_h R_{12} S_{yxh} + 2g_h R_{13} S_{yz} - 2g_h^2 R_{12} R_{13} S_{xz}\right] \quad (9)$$

where,

$$R_{12} = \frac{\bar{Y}}{\bar{X}_{1h}} \text{ and } R_{13} = \frac{\bar{Y}}{\bar{Z}_{1h}}$$

***Proposed estimator under stratified sampling scheme***

Motivated by [11], we propose the following hybrid exponential type regression-cum-ratio estimators of finite population mean in stratified random sampling under two conditions.

***Case I: Proposed estimator under stratified sampling when auxiliary variables are not transformed***

The proposed estimator when the variables are not transformed is given as:

$$\bar{y}_{uv.st} = [\bar{y}_{st} + \beta_h(\bar{Z}_{st} - \bar{z}_{st})]exp\left[\alpha_h\left(\frac{\bar{X}_{st} - \bar{x}_{st}}{\bar{X}_{st} + \bar{x}_{st}}\right)\right] \quad (10)$$

where, $\beta_h$ and $\alpha_h$ are suitably constants which minimize the MSE of $\bar{y}_{uv.st}$.

We define:

$\bar{y}_h = \bar{Y}_h(1 + e_{y,h})$; $\bar{x}_h = \bar{X}_h(1 + e_{x,h})$; $\bar{z}_h = \bar{Z}_h(1 + e_{z,h})$; $E(e_{y,h}) = E(e_{x,h}) = E(e_{z,h}) = 0$, $E(e_{y,h}^2) = \sum_{h=1}^{L} W_h^2\,\theta_h C_{y,h}^2$; $E(e_{x,h}^2) = \sum_{h=1}^{L} W_h^2\,\theta_h C_{x,h}^2$; $E(e_{z,h}^2) = \sum_{h=1}^{L} W_h^2\,\theta_h C_{z,h}^2$; $E(e_{y,h}e_{x,h}) = \sum_{h=1}^{L} W_h^2\,\theta_h \rho_{y,x,h} C_{y,h} C_{x,h}$; $E(e_{y,h}e_{z,h}) = \sum_{h=1}^{L} W_h^2\,\theta_h C_{y,h} C_{z,h}$; $E(e_{x,h}e_{z,h}) = \sum_{h=1}^{L} W_h^2\,\theta_h \rho_{x,z,h} C_{x,h} C_{z,h}$

where,

$\theta_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$; $h = 1, 2, \dots, L$, is the sampling fraction of the $h^{th}$ stratum.

***Theorem I:***

The bias of the alternative hybrid regression-cum-ratio exponential type estimator of finite population mean under SRS for stratification when the auxiliary variables are not transformed is given by:

$$Bias(\bar{y}_{uv.st}) = \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\,\theta_h\left(\frac{1}{8}\alpha_h^2 C_{xh}^2 - \frac{1}{2}\alpha_h \rho_{yxh} C_{xh} C_{yh}\right) + \frac{1}{2}\sum_{h=1}^{L} W_h^2\,\beta_h \bar{Z}_{st}\alpha_h\theta_h\rho_{xzh} C_{xh} C_{zh} \quad (11)$$

***Proof of Theorem I:***

Consider the expression in equation (10) above. Inserting the appropriate definitions of section 2 into the following expression, we have:

$$\bar{y}_{uv.st} = [\bar{y}_{st} + \beta_h(\bar{Z}_{st} - \bar{z}_{st})]exp\left[\alpha_h\left(\frac{\bar{X}_{st} - \bar{x}_{st}}{\bar{X}_{st} + \bar{x}_{st}}\right)\right]$$

$$= \left[\sum_{h=1}^{L} W_h \bar{y}_h + \beta_h \left(\sum_{h=1}^{L} W_h \bar{Z}_h - \sum_{h=1}^{L} W_h \bar{z}_h\right)\right] * exp\left[\alpha_h \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h - \sum_{h=1}^{L} W_h \bar{x}_h}{\sum_{h=1}^{L} W_h \bar{X}_h + \sum_{h=1}^{L} W_h \bar{x}_h}\right)\right]$$

$$= \left[\sum_{h=1}^{L} W_h \bar{Y}_h (1 + e_{yh}) + \beta_h \left(\sum_{h=1}^{L} W_h \bar{Z}_h - \sum_{h=1}^{L} W_h \bar{Z}_h (1 + e_{zh})\right)\right] * exp\left[\alpha_h \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h - \sum_{h=1}^{L} W_h \bar{X}_h (1 + e_{xh})}{\sum_{h=1}^{L} W_h \bar{X}_h + \sum_{h=1}^{L} W_h \bar{X}_h (1 + e_{xh})}\right)\right]$$

$$= \left[\sum_{h=1}^{L} W_h \bar{Y}_h (1 + e_{yh}) + \beta_h \left(\sum_{h=1}^{L} W_h \bar{Z}_h - \sum_{h=1}^{L} W_h \bar{Z}_h (1 + e_{zh})\right)\right] * exp\left\{\alpha_h \left[\frac{\sum_{h=1}^{L} W_h \bar{X}_h \left(1 - (1 + e_{xh})\right)}{\sum_{h=1}^{L} W_h \bar{X}_h \left(1 + (1 + e_{xh})\right)}\right]\right\}$$

$$= \sum_{h=1}^{L} W_h \left[\bar{Y}_h (1 + e_{yh}) - \beta_h Z_h e_{zh}\right] exp\left[\alpha_h \left(\frac{-e_{xh}}{2 + e_{xh}}\right)\right]$$

$$= \sum_{h=1}^{L} W_h \left[\bar{\bar{Y}}_h + \bar{Y} e_{yh} - \beta_h Z_h e_{zh}\right] exp\left[\frac{-\alpha_h e_{xh}}{2}\left(1 + \frac{e_{xh}}{2}\right)^{-1}\right]$$

$$= \sum_{h=1}^{L} W_h \left[\bar{\bar{Y}}_h + \bar{Y} e_{yh} - \beta_h Z_h e_{zh}\right] * exp\left[-\frac{1}{2}\alpha_h e_{xh}\left(1 - \frac{1}{2} e_{xh} + \frac{1}{4} e_{xh}^2 - \cdots\right)\right]$$

$$= \sum_{h=1}^{L} W_h \left[\bar{\bar{Y}}_h + \bar{Y} e_{yh} - \beta_h Z_h e_{zh}\right] exp\left[-\frac{1}{2}\alpha_h e_{xh} + \frac{1}{4}\alpha_h e_{xh}^2 - \frac{1}{8}\alpha_h e_{xh}^3 + \cdots\right]$$

By First order approximation principle, we have:

$$= \sum_{h=1}^{L} W_h \left[\bar{\bar{Y}}_h + \bar{Y} e_{yh} - \beta_h Z_h e_{zh}\right] exp\left[-\frac{1}{2}\alpha_h e_{xh}\right] \tag{12}$$

Expanding the exponential part of the expression in (12), we have

$$= \sum_{h=1}^{L} W_h \left[\bar{\bar{Y}}_h + \bar{Y} e_{yh} - \beta_h \bar{Z}_h e_{zh}\right]\left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{2!}\cdot\frac{1}{2^2}\alpha_h^2 e_{xh}^2 - \frac{1}{3!}\cdot\frac{1}{2^3}\alpha_h^3 e_{xh}^3 + \cdots\right]$$

$$= \sum_{h=1}^{L} W_h \left[\bar{Y}_h + \bar{Y}_h e_{yh} - \beta_h \bar{Z}_h e_{zh}\right]\left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 - \frac{1}{48}\alpha_h^3 e_{xh}^3 + \cdots\right]$$

$$= \sum_{h=1}^{L} W_h \left[\bar{Y}_h + \bar{Y}_h e_{yh} - \beta_h \bar{Z}_h e_{zh}\right]\left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right]$$

$$= \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h \left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right] + \bar{Y}_h e_{yh} \left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right] - \beta_h \bar{Z}_h e_{zh} \left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right]\right\}$$

$$= \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h \left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right] + \bar{Y}_h \left[e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh} + \frac{1}{8}\alpha_h^2 e_{xh}^2\right] - \beta_h \bar{Z}_h \left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 e_{zh}\right]\right\}$$

$$= \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h \left[1 - \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 + e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 e_{yh}\right] - \beta_h \bar{Z}_h \left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 e_{zh}\right]\right\}$$

$$= \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h + \bar{Y}_h \left[- \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 + e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh}\right] - \beta_h \bar{Z}_h \left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh}\right]\right\}$$

$$\bar{y}_{uv.st} - \bar{Y}_{st} = \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h \left[- \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 + e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh}\right] - \beta_h \bar{Z}_h \left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh}\right]\right\} \quad (13)$$

Taking expectation on both sides of (13), and inserting the definitions of expectations above, we have:

$$Bias(\bar{y}_{uv}) = E(\bar{y}_{uv.st} - \bar{Y}_{st})$$

$$= \sum_{h=1}^{L} W_h \left\{ \bar{Y}_h E\left[- \frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 + e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh}\right] - \beta_h \bar{Z}_h E\left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh}\right]\right\}$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h \left[-\frac{1}{2}\alpha_h E(e_{xh}) + \frac{1}{8}\alpha_h^2 E(e_{xh}^2) + E(e_{yh}) - \frac{1}{2}\alpha_h E(e_{xh}e_{yh})\right] - \beta_h \sum_{h=1}^{L} W_h \bar{Z}_h \left[E(e_{zh}) - \frac{1}{2}\alpha_h E(e_{xh}e_{zh})\right]$$

$$= \bar{Y}_{st}\left[0 + \frac{1}{8}\sum_{h=1}^{L} W_h^2\, \alpha^2 \theta_h C_{xh}^2 + 0 - \frac{1}{2}\sum_{h=1}^{L} W_h^2\, \alpha_h \theta_h \rho_{yxh} C_{xh} C_{yh}\right] - \sum_{h=1}^{L} W_h^2\, \beta_h \bar{Z}_{st}\left[0 - \frac{1}{2}\alpha_h \theta_h \rho_{xzh} C_{xh} C_{zh}\right]$$

$$= \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\, \theta_h \left(\frac{1}{8}\alpha_h^2 C_{xh}^2 - \frac{1}{2}\alpha_h \rho_{yxh} C_{xh} C_{yh}\right) + \frac{1}{2}\sum_{h=1}^{L} W_h^2\, \beta_h \bar{Z}_{st}\alpha_h \theta_h \rho_{xzh} C_{xh} C_{zh}$$

Therefore,

$$Bias(\bar{y}_{uv.st}) = \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\, \theta_h \left(\frac{1}{8}\alpha_h^2 C_{xh}^2 - \frac{1}{2}\alpha_h \rho_{y,x,h} C_{x,h} C_{y,h}\right) + \frac{1}{2}\sum_{h=1}^{L} W_h^2\, \beta_h \bar{Z}_{st}\alpha_h \theta_h \rho_{x,z,h} C_{x,h} C_{z,h}$$

***Theorem 2:***
The Mean Square Error (MSE) of the alternative hybrid regression-ratio exponential type estimator finite population mean under simple random sampling without replacement for stratified sampling is given by:

$$MSE(\bar{y}_{uv.st}) = \bar{Y}_{st}^2 \sum_{h=1}^{L} \frac{W_h^2 C_{yh}^2 \theta_h}{\rho_{x,z,h}^2 - 1}\left(\rho_{y,x,h}^2 + \rho_{x,z,h}^2 + \rho_{y,z,h}^2 - 2\rho_{y,x,h}\rho_{x,z,h}\rho_{y,z,h} - 1\right); \quad \rho_{y,x,h}, \rho_{x,z,h}, \rho_{y,z,h} \neq \pm 1 \qquad (14)$$

**Proof of Theorem 2:**
$MSE(\bar{y}_{uv}) = E(\bar{y}_{uv.st} - \bar{Y}_{st})^2$, therefore, squaring both sides of (13), we have:

$$(\bar{y}_{uv.st} - \bar{Y}_{st})^2 = \left[\sum_{h=1}^{L} W_h \left\{\bar{Y}_h \left[-\frac{1}{2}\alpha_h e_{xh} + \frac{1}{8}\alpha_h^2 e_{xh}^2 + e_{yh} - \frac{1}{2}\alpha_h e_{xh} e_{yh}\right] - \beta_h \bar{Z}_h \left[e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh}\right]\right\}\right]^2$$

$$= \left[\sum_{h=1}^{L} W_h \bar{Y}_h \left(e_{yh} - \frac{1}{2}\alpha_h e_{xh}\right) - \beta_h \sum_{h=1}^{L} W_h \bar{Z}_h e_{zh}\right]^2$$

$$= \left[\sum_{h=1}^{L} W_h^2\, \bar{Y}_h^2 \left(e_{yh} - \frac{1}{2}\alpha_h e_x\right)^2 - 2\left(\beta_h \sum_{h=1}^{L} W_h \bar{Z}_h e_{zh}\right) \sum_{h=1}^{L} W_h\, \bar{Y}_h \left(e_{yh} - \frac{1}{2}\alpha_h e_{xh}\right) + \beta_h^2 \sum_{h=1}^{L} W_h^2\, \bar{Z}_h^2 e_{zh}^2\right]$$

$$= \bar{Y}_{st}^2 \left(e_{yh}^2 - 2.\frac{1}{2}\alpha_h e_{xh} e_{yh} + \frac{1}{4}\alpha_h^2 e_{xh}^2\right) - 2\, \beta_h \bar{Y}_{st} \bar{Z}_{st} \left(e_{yh} e_{zh} - \frac{1}{2}\alpha_h e_{xh} e_{zh}\right) + \beta_h^2 \bar{Z}_{st}^2 e_z^2$$

$$E(\bar{y}_{uv.st} - \bar{Y}_{st})^2 = \bar{Y}_{st}^2 \left(E\left(e_{yh}^2\right) - \alpha_h E\left(e_{xh} e_{yh}\right) + \frac{1}{4}\alpha_h^2 E(e_{xh}^2)\right) -2\, \beta_h \bar{Y}_{st} \bar{Z}_{st} \left[E\left(e_{yh} e_{zh}\right) - \frac{1}{2}\alpha_h E(e_{xh} e_{zh})\right] + \beta_h^2 \bar{Z}_{st}^2 E(e_{zh}^2)$$

$$MSE(\bar{y}_{uv.st}) = E(\bar{y}_{uv.st} - \bar{Y}_{st})^2$$

$$= \bar{Y}_{st}^2 \sum_{h=1}^{L} W_h^2 \left(\theta_h C_{yh}^2 - \alpha_h \theta_h \rho_{yxh} C_{xh} C_{yh} + \frac{1}{4}\alpha_h^2 \theta_h C_{xh}^2\right) -2\bar{Z}_{st} \sum_{h=1}^{L} W_h^2\, \beta_h \bar{Y}_{st} \left(\theta_h \rho_{yzh} C_{yh} C_{zh} - \frac{1}{2}\alpha_h \theta_h \rho_{xz} C_{xh} C_{zh}\right) + \bar{Z}_{st}^2 \sum_{h=1}^{L} W_h^2\, \beta_h^2 \theta_h C_{zh}^2 \quad (15)$$

To obtain the optimal value of $\alpha_h$and$\beta_h$that minimizes the $MSE(\bar{y}_{uv.st})$, we differentiate (15) with respect to $\alpha_h, \beta_h$and equate to zero.

$$\frac{\partial MSE(\bar{y}_{uv.st})}{\partial \alpha_h} = \frac{\partial}{\partial \alpha}\left\{\bar{Y}_{st}^2 \left(\theta_h C_{yh}^2 - \alpha_h \theta_h \rho_{yxh} C_{xh} C_{yh} + \frac{1}{4}\alpha_h^2 \theta_h C_{xh}^2\right) -2\, \beta_h \bar{Y}_{st} \bar{Z}_{st} \left(\theta_h \rho_{yzh} C_{yh} C_{zh} - \frac{1}{2}\alpha_h \theta_h \rho_{xz} C_{xh} C_{zh}\right) + \beta_h^2 \bar{Z}_{st}^2 \theta_h C_{zh}^2\right\} = 0$$

$$\Rightarrow \bar{Y}_{st}^2 \left(\frac{1}{2}\alpha_h \theta_h C_{xh}^2 - \theta_h \rho_{yxh} C_{xh} C_{yh}\right) + \beta_h \bar{Y}_{st} \bar{Z}_{st} \theta_h \rho_{xzh} C_{xh} C_{zh} = 0$$

$$\bar{Y}_{st}^2 \left(\frac{1}{2}\alpha_h \theta_h C_{xh}^2 - \theta_h \rho_{yxh} C_{xh} C_{yh}\right) = -\, \beta_h \bar{Y}_{st} \bar{Z}_{st} \theta_h \rho_{xzh} C_{xh} C_{zh}$$

$$\frac{\alpha_h}{2} = \frac{\bar{Y}_{st} \rho_{yxh} C_{xh} C_{yh} - \beta_h \bar{Z}_h \rho_{xzh} C_{xh} C_{zh}}{\bar{Y}_{st} C_{xh}^2}$$

$$\alpha_h = \frac{2\left(\bar{Y}_{st} \rho_{yxh} C_{yh} - \beta_h \bar{Z}_{st} \rho_{xzh} C_{zh}\right)}{\bar{Y}_{st} C_{xh}} \quad (16)$$

$$\frac{\partial MSE(\bar{y}_{uv.st})}{\partial \beta_h} = 2\bar{Y}_{st}\bar{Z}_{st}\left(\frac{1}{2}\alpha_h\theta_h\rho_{xzh}C_{xh}C_{zh} - \theta_h\rho_{yzh}C_{yh}C_{zh}\right) + 2\beta_h\bar{Z}_{st}^2C_{zh}^2\theta_h$$

$$\bar{Y}_{st}\bar{Z}_{st}\left(\frac{1}{2}\alpha_h\theta_h\rho_{xzh}C_{xh}C_{zh} - \theta_h\rho_{yzh}C_{yh}C_{zh}\right) + \beta_h\bar{Z}_{st}^2C_{zh}^2\theta_h = 0$$

$$\beta_h\bar{Z}_{st}C_{zh}\theta_h = \bar{Y}_{st}\left(\theta_h\rho_{yzh}C_{yh} - \frac{1}{2}\alpha_h\theta_h\rho_{xzh}C_{xh}\right)$$

$$\beta_h = -\frac{1}{2}\frac{\bar{Y}_{st}\left(\alpha_h\rho_{xzh}C_{xh} - 2\rho_{yzh}C_{yh}\right)}{\bar{Z}_{st}C_{zh}} \qquad (17a)$$

Putting (16) into (17a):

$$\beta_h = -\frac{\bar{Y}_{st}\rho_{yxh}\rho_{xzh}C_{yh} - \beta_h\bar{Z}_{st}\rho_{xzh}^2C_{zh} - \bar{Y}_{st}\rho_{yzh}C_{yh}}{\bar{Z}_{st}C_{zh}}$$

$$-\beta_h\bar{Z}_{st}C_{zh} = \bar{Y}_{st}\rho_{yxh}\rho_{xzh}C_{yh} - \beta_h\bar{Z}_{st}\rho_{xzh}^2C_{zh} - \bar{Y}_{st}\rho_{yzh}C_{yh}$$

$$\beta_h\bar{Z}_{st}\rho_{xzh}^2C_{zh} - \beta_h\bar{Z}_{st}C_{zh} = \bar{Y}_{st}\rho_{yxh}\rho_{xzh}C_{yh} - \bar{Y}_{st}\rho_{yzh}C_{yh}$$

$$\beta_h = \frac{\bar{Y}_{st}\left(\rho_{yxh}\rho_{xzh}C_{yh} - \rho_{yzh}C_{yh}\right)}{\bar{Z}_{st}\rho_{xzh}^2C_{zh} - \bar{Z}_{st}C_{zh}} \qquad (17b)$$

Substituting the value of (16) and (17b) into Equation (15), gives on simplification:

$$MSE(\bar{y}_{uv.st}) = \sum_{h=1}^{L} W_h^2\,\theta_h(\bar{Y}_{st}^2C_{yh}^2 - \bar{Y}_{st}^2C_{yhh}^2\rho_{yxh}^2 + 2\beta_h\bar{Y}_{st}\bar{Z}_{st}\rho_{xzh}\rho_{yxh}C_{yh}C_{zh}$$
$$-2\beta_h\bar{Y}_{st}\bar{Z}_{st}\rho_{yzh}C_{yh}C_{zh} - \beta_h^2\bar{Z}_{st}^2\rho_{xzh}^2C_{zh}^2 - \beta_h^2\bar{Z}_{st}^2C_{zh}^2)$$
$$= \sum_{h=1}^{L} W_h^2\,\frac{\bar{Y}_{st}^2C_{yh}^2\theta_h\left(\rho_{yxh}^2 + \rho_{xzh}^2 + \rho_{yzh}^2 - 2\rho_{yxh}\rho_{xzh}\rho_{yzh} - 1\right)}{\rho_{xzh}^2 - 1}$$

Therefore,

$$MSE(\bar{y}_{uv.st}) = \bar{Y}_{st}^2\sum_{h=1}^{L}\frac{W_h^2C_{y,h}^2\theta_h}{\rho_{x,z,h}^2 - 1}\left(\rho_{y,x,h}^2 + \rho_{x,z,h}^2 + \rho_{y,z,h}^2\right.$$
$$\left.-2\rho_{y,x,h}\rho_{x,z,h}\rho_{y,z,h} - 1\right);\ \rho_{xzh}, \rho_{yzh}, \rho_{yxh} \neq \pm 1$$

We now, substitute the values of $\alpha_h$ and $\beta_h$ from (16) and (17b) respectively, into (11) to obtain bias of $\bar{y}_{uv.st}$ as:

$$Bias(\bar{y}_{uv.st}) = \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\,\theta_h\left(\frac{1}{8}\alpha_h^2C_{xh}^2 - \frac{1}{2}\alpha_h\rho_{y,x,h}C_{x,h}C_{y,h}\right)$$
$$+\frac{1}{2}\sum_{h=1}^{L} W_h^2\,\beta_h\bar{Z}_{st}\alpha_h\theta_h\rho_{x,z,h}C_{x,h}$$

$$\therefore Bias(\bar{y}_{uv.st}) = -\frac{1}{2}\bar{Y}_{st}\sum_{h=1}^{L} W_h^2\frac{C_{yh}^2\theta_h}{(\rho_{xzh}^2 - 1)^2}\left(\rho_{xzh}\rho_{yzh} - \rho_{yxh}\right)^2,\quad \rho_{xzh} \neq 1 \qquad (18)$$

***Case II: Proposed estimator in stratified random sampling when auxiliary variables are transformed.***

The proposed estimator when the variables are transformed is given as:

$$\bar{y}_{tv.st} = [\bar{y}_{st} + \beta_{1h}(\bar{z}_{st}^* - \bar{Z}_{st})]\exp\left[\alpha_{1h}\left(\frac{\bar{x}_{st}^* - \bar{X}_{st}}{\bar{X}_{st} + \bar{x}_{st}^*}\right)\right] \qquad (19)$$

where, $\beta_{1h}$ and $\alpha_{1h}$suitably chosen constants that minimize MSE of $\bar{y}_{tv.st}$.

***Theorem 3***

The bias of the hybrid regression-cum-ratio exponential type estimator using SRSWOR, under stratified random sampling when the auxiliary variables are transformed is given by:

$$Bias(\bar{y}_{tv.st}) = \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\left[-\frac{1}{2}\alpha_{1h}g_h\theta_h\rho_{yxh}C_{xh}C_{yh} + \frac{1}{8}\alpha_{1h}^2 g_h^2\theta_h C_{xh}^2\right] + \frac{1}{2}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\beta_{1h}\,\alpha_{1h}g_h^2\theta_h\rho_{xzh}C_{xh}C_{zh} \qquad (20)$$

***Proof:***

Consider the expression in (19)

$$\bar{y}_{tv.st} = [\bar{y}_{st} + \beta_{1h}(\bar{z}_{st}^{*} - \bar{Z}_{st})]\exp\left[\alpha_{1h}\left(\frac{\bar{x}_{st}^{*} - \bar{X}_{st}}{\bar{X}_{st} + \bar{x}_{st}^{*}}\right)\right]$$

By substituting the basic definitions for $\bar{y}_{st}$, $\bar{x}_{st}^{*}$ and $\bar{z}_{st}^{*}$, we have:

$$\bar{y}_{tv.st} = \sum_{h=1}^{L} W_h\left[(1+e_{yh})\bar{Y}_h + \beta_{1h}(1-g_he_{zh})\bar{Z}_h - \bar{Z}_h\right] * \exp\left\{\alpha_{1h}\left[\frac{\sum_{h=1}^{L} W_h\left((1-g_he_{xh})\bar{X}_h - \bar{X}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + (1-g_he_{xh})\bar{X}_h\right)}\right]\right\}$$

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h]\exp\left[\alpha_{1h}\left(\frac{-g_he_{xh}}{2-g_he_{xh}}\right)\right]$$

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h]\exp\left[-\frac{1}{2}\alpha_{1h}g_he_{xh}\left(1-\frac{1}{2}g_he_{xh}\right)^{-1}\right]$$

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h] * \exp\left[-\frac{1}{2}\alpha_{1h}g_he_{xh}\left(1+\frac{1}{2}g_he_{xh}+\frac{1}{4}g_h^2e_{xh}^2+\frac{1}{8}g_h^3e_{xh}^3 + \cdots\right)\right]$$

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h] * \exp\left[-\frac{1}{2}\alpha_{1h}g_he_{xh}+\frac{1}{4}\alpha_{1h}g_h^2e_{xh}^2-\frac{1}{8}\alpha_{1h}g_h^3e_{xh}^3 + \cdots\right]$$

By first order approximation,

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h]\exp\left[-\frac{1}{2}\alpha_{1h}g_he_{xh}\right] \qquad (21)$$

Expanding the exponential part of (21) we have:

$$= [\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_he_{zh}\bar{Z}_h]\left[1-\frac{1}{2}\alpha_{1h}g_he_{xh}+\frac{1}{2!}\left(\frac{1}{2}\alpha_{1h}g_he_{xh}\right)^2 - \frac{1}{3!}\left(\frac{1}{2}\alpha_{1h}g_he_{xh}\right)^3 + \cdots\right]$$

$$= \left[\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_h e_{zh}\bar{Z}_h\right]\left[1 - \frac{1}{2}\alpha_{1h}g_h e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_h^2 e_{xh}^2 - \frac{1}{48}\alpha_{1h}^3 g_h^3 e_{xh}^3 + \cdots\right]$$

$$= \left[\bar{Y}_{st} + \bar{Y}_{st}e_{yh} - \beta_{1h}g_h e_{zh}\bar{Z}_h\right]\left[1 - \frac{1}{2}\alpha_{1h}g_h e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_h^2 e_{xh}^2\right]$$

$$= \bar{Y}_{st} + \bar{Y}_{st}e_{yh}\left[1 - \frac{1}{2}\alpha_{1h}g_h e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_h^2 e_{xh}^2\right] - \beta_{1h}g_{1h}e_{zh}\bar{Z}_h\left[1 - \frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2 e_{xh}^2\right]$$

$$= \bar{Y}_{st} + \bar{Y}_{st}\left[-\frac{1}{2}\alpha_{1h}g_h e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_h^2 e_{xh}^2\right] + \bar{Y}_{st}\left[e_{yh} - \frac{1}{2}\alpha_{1h}g_h e_{xh}e_{yh} + \frac{1}{8}\alpha_{1h}^2 g_h^2 e_{xh}^2 e_{yh}\right] - \beta_{1h}\bar{Z}_h\left[g_{1h}e_{zh} - \frac{1}{2}\alpha_1 g_{1h}^2 e_{xh}e_{zh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^3 e_{xh}^2 e_{zh}\right]$$

$$\bar{y}_{tv.st} - \bar{Y}_{st} = \bar{Y}_{st}\left[-\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2 e_{xh}^2\right] + \bar{Y}_{st}\left[e_{yh} - \frac{1}{2}\alpha_{1h}g_{1h}e_{xh}e_{yh}\right] - \beta_{1h}\bar{Z}_h\left[g_{1h}e_{zh} - \frac{1}{2}\alpha_1 g_{1h}^2 e_{xh}e_{zh}\right]$$

$$\bar{y}_{tv.st} - \bar{Y}_{st} = \bar{Y}_{st}\left[-\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + e_{yh} - \frac{1}{2}\alpha_{1h}g_{1h}e_{xh}e_{yh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2 e_{xh}^2\right] - \beta_{1h}\bar{Z}_{st}\left[g_{1h}e_{zh} - \frac{1}{2}\alpha_{1h}g_{1h}^2 e_{xh}e_{zh}\right] \quad (22)$$

Taking expectation on both sides of (22) gives us the bias.

$$Bias(\bar{y}_{tv.st}) = E(\bar{y}_{tv.st} - \bar{Y}_{st})$$

$$= \bar{Y}_{st}\left[-\frac{1}{2}\alpha_{1h}g_{1h}E(e_{xh}) + E(e_{yh}) - \frac{1}{2}\alpha_{1h}g_{1h}E(e_{xh}e_{yh}) + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2 E(e_{xh}^2)\right] - \beta_{1h}\bar{Z}_{st}\left[g_{1h}E(e_{zh}) - \frac{1}{2}\alpha_{1h}g_{1h}^2 E(e_{xh}e_{zh})\right]$$

$$= \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\left[0 + 0 - \frac{1}{2}\alpha_{1h}g_{1h}\theta_{1h}\rho_{yxh}C_{xh}C_{yh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2\theta_{1h}C_{xh}^2\right] - \beta_{1h}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\left[0 - \frac{1}{2}\alpha_{1h}g_{1h}^2\theta_{1h}\rho_{xzh}C_{xh}C_{zh}\right]$$

$$= \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\left[-\frac{1}{2}\alpha_{1h}g_{1h}\theta_{1h}\rho_{yxh}C_{xh}C_{yh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2\theta_{1h}C_{xh}^2\right] + \beta_{1h}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\left[\frac{1}{2}\alpha_{1h}g_{1h}^2\theta_{1h}\rho_{xzh}C_{xh}C_{zh}\right]$$

Therefore,

$$Bias(\bar{y}_{tv}) = \bar{Y}_{st}\sum_{h=1}^{L} W_h^2\left[\frac{1}{8}\alpha_{1h}^2 g_{1h}^2\theta_{1h}C_{xh}^2 - \frac{1}{2}\alpha_{1h}g_{1h}\theta_{1h}\rho_{yxh}C_{xh}C_{yh}\right] + \frac{1}{2}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\,\beta_{1h}\alpha_{1h}g_{1h}^2\theta_{1h}\rho_{xzh}C_{xh}C_{zh}$$

***Theorem 4***

The MSE of the hybrid regression–ratio exponential type estimator in stratified random sampling is given by:

$$MSE(\bar{y}_{tv.st}) = \bar{Y}_{st}^2\sum_{h=1}^{L} W_h^2\frac{C_{yh}^2\theta_{1h}}{\rho_{xzh}^2 - 1}\left[\rho_{yzh}^2 + \rho_{xzh}^2 + \rho_{yxh}^2 - 2\rho_{yxh}\rho_{xzh}\rho_{yzh} - 1\right] \quad (23)$$

where, $\rho_{xzh}, \rho_{xzh}, \rho_{yzh} \pm 1$ .

***Proof:***

In order to obtain the MSE of $\bar{y}_{tp.st}$, we square both sides of Equation (22) and take expectations.

$$(\bar{y}_{tv.st} - \bar{Y}_{st})^2 = \left[\bar{Y}_{st}\left(-\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + e_{yh} - \frac{1}{2}\alpha_{1h}g_{1h}e_{xh}e_{yh} + \frac{1}{8}\alpha_{1h}^2 g_{1h}^2 e_{xh}^2\right) - \beta_{1h}\bar{Z}_{st}\left(g_{1h}e_{zh} - \frac{1}{2}\alpha_{1h}g_{1h}^2 e_{xh}e_{zh}\right)\right]^2$$

$$= \left[\bar{Y}_{st}\left(-\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + e_{yh}\right) - \beta_{1h}\bar{Z}_{st}(g_{1h}e_{zh})\right]^2$$

$$= \bar{Y}_{st}^2\left(-\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + e_{yh}\right)^2 - 2\beta_{1h}\bar{Y}_{st}\bar{Z}_{st}g_{1h}e_{zh}\left(\frac{1}{2}\alpha_{1h}g_{1h}e_{xh} + e_{yh}\right) + \beta_{1h}^2\bar{Z}_{st}^2 g_{1h}^2 e_{zh}^2$$

$$= \bar{Y}_{st}^2\left[\frac{1}{4}\alpha_{1h}^2 g_{1h}^2 e_{xh}^2 - \alpha_{1h}g_{1h}e_{xh}e_{yh} + e_{yh}^2\right] - 2\,\beta_{1h}\bar{Y}_{st}\bar{Z}_{st}\left[\frac{1}{2}\alpha_{1h}g_{1h}^2 e_{xh}e_{zh} + g_{1h}e_{yh}e_{zh}\right] + \beta_{1h}^2\bar{Z}_{st}^2 g_{1h}^2 e_{zh}^2 \quad (24)$$

Taking expectations on both sides of (24) and substituting the definition of expectations, we have:

$$E(\bar{y}_{tv.st} - \bar{Y}_{st})^2 = \bar{Y}_{st}^2\left[\frac{1}{4}\alpha_{1h}^2 g_{1h}^2 E(e_{xh}^2) - \alpha_{1h}g_{1h}E(e_{xh}e_{yh}) + E(e_{yh}^2)\right] - 2\,\beta_{1h}\bar{Y}_{st}\bar{Z}_{st}\left[\frac{1}{2}\alpha_{1h}g_{1h}^2 E(e_{xh}e_{zh}) + g_{1h}E(e_{yh}e_{zh})\right] + \beta_{1h}^2\bar{Z}_{st}^2 g_{1h}^2 E(e_{zh}^2)$$

$$= \bar{Y}_{st}^2\sum_{h=1}^{L} W_h^2\left(\frac{1}{4}\alpha_{1h}^2 g_{1h}^2\theta_{1h}C_{xh}^2 - \alpha_{1h}g_{1h}\theta_{1h}\rho_{yxh}C_{xh}C_{yh} + \theta_{1h}C_{yh}^2\right) - 2\bar{Y}_{st}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\,\beta_{1h}\left(\frac{1}{2}\alpha_{1h}g_{1h}^2\theta_{1h}\rho_{xzh}C_{xh}C_{zh} + g_{1h}\theta_{1h}\rho_{yzh}C_{yh}C_{zh}\right) + \bar{Z}_{st}^2\sum_{h=1}^{L} W_h^2\,\beta_{1h}^2 g_{1h}^2\theta_{1h}C_{zh}^2$$

Therefore,

$$MSE(\bar{y}_{tv.st}) = E(\bar{y}_{tv.st} - \bar{Y}_{st})^2$$

$$= \bar{Y}_{st}^2 \sum_{h=1}^{L} W_h^2 \left(\frac{1}{4}\alpha_{1h}^2 g_{1h}^2 \theta_{1h} C_{xh}^2 - \alpha_{1h} g_{1h} \theta_{1h} \rho_{yxh} C_{xh} C_{yh} + \theta_{1h} C_{yh}^2\right)$$
$$- 2\,\bar{Y}_{st}\bar{Z}_{st} \sum_{h=1}^{L} W_h^2\, \beta_{1h} \left(-\frac{1}{2}\alpha_{1h} g_{1h}^2 \theta_{1h} \rho_{xzh} C_{xh} C_{zh} + g_{1h}\theta_{1h}\rho_{yzh} C_{yh} C_{zh}\right)$$
$$+ \bar{Z}_{st}^2 \sum_{h=1}^{L} W_h^2\, \beta_{1h}^2 g_{1h}^2 \theta_{1h} C_{zh}^2 \qquad (25)$$

To obtain the value of $\alpha_{1h}$, $\beta_{1h}$ that minimizes the MSE, we differentiate Equation (25) partially with respect to $\alpha_{1,h}, \beta_{1,h}$ and equate to zero:

$$\frac{\partial MSE(\bar{y}_{tv.st})}{\partial \alpha_{1,h}} = \frac{\partial}{\partial \alpha_{1,h}} \left[ \bar{Y}_{st}^2 \sum_{h=1}^{L} W_h^2 \left(\frac{1}{4}\alpha_{1,h}^2 g_{1,h}^2 \theta_{1,h} C_{x,h}^2 - \alpha_{1,h} g_{1,h}\theta_{1,h}\rho_{y,x,h} C_{x,h} C_{y,h} + \theta_{1,h} C_{y,h}^2\right) - 2\,\bar{Y}_{st}\bar{Z}_{st} \sum_{h=1}^{L} W_h^2\, \beta_{1,h} \left(-\frac{1}{2}\alpha_{1,h} g_{1,h}^2 \theta_{1,h}\rho_{x,z,h} C_{x,h} C_{z,h} + g_{1,h}\theta_{1,h}\rho_{y,z,h} C_{y,h} C_{z,h}\right) + \bar{Z}_{st}^2 \sum_{h=1}^{L} W_h^2\, \beta_{1,h}^2 g_{1,h}^2 \theta_{1,h} C_{z,h}^2 \right] = 0$$

$$\bar{Y}_{st}^2 \left(2.\frac{1}{4}\alpha_{1,h} g_{1,h}^2 \theta_{1,h} C_{x,h}^2 - g_{1,h}\theta_{1,h}\rho_{y,x,h} C_{x,h} C_{y,h}\right) - 2\beta_{1,h}\bar{Y}_{st}\bar{Z}_{st}\left(\frac{1}{2} g_{1,h}^2 \theta_{1,h}\rho_{x,z,h} C_{x,h} C_{z,h}\right) = 0$$

$$\bar{Y}_{st}^2 \left(\frac{1}{2}\alpha_{1,h} g_{1,h}^2 \theta_{1,h} C_{x,h}^2 - g_{1,h}\theta_{1,h}\rho_{y,x,h} C_{x,h} C_{y,h}\right) + \beta_{1,h}\bar{Y}_{st}\bar{Z}_{st} g_{1,h}^2 \theta_{1,h}\rho_{x,z,h} C_{x,h} C_{z,h} = 0$$

$$\frac{1}{2}\alpha_{1h} g_{1h}^2 C_{xh} = \frac{\bar{Y}_{st} g_{1h}\rho_{yxh} C_{yh} - \beta_{1h}\bar{Z}_{st} g_{1h}^2 \rho_{xzh} C_{zh}}{\bar{Y}_{st}}$$

$$\alpha_{1,h} = \frac{\bar{Y}_{st} g_{1,h}\rho_{y,x,h} C_{y,h} - 2(\beta_{1,h}\bar{Z}_{st} g_{1,h}^2 \rho_{x,z,h} C_{z,h})}{\bar{Y}_{st} g_{1,h}^2 C_{x,h}}$$

Therefore,

$$\alpha_{1h} = \frac{2\left(\bar{Y}_{st}\rho_{y,x,h} C_{y,h} - \beta_{1,h}\bar{Z}_{st} g_{1,h}\rho_{x,z} C_z\right)}{\bar{Y}_{st} g_{1,h} C_{x,h}} \qquad (26a)$$

$$\frac{\partial MSE(\bar{y}_{tv.st})}{\partial \beta_{1,h}} = -2\bar{Y}_{st}\bar{Z}_{st}\left(-\frac{1}{2}\alpha_{1,h} g_{1,h}^2 \theta_{1,h}\rho_{x,z,h} C_{x,h} C_{z,h} + g_{1,h}\theta_{1,h}\rho_{y,z,h} C_{y,h} C_{z,h}\right) + 2\,\beta_{1,h}\bar{Z}_{st}^2 g_{1,h}^2 \theta_{1,h} C_{z,h}^2 = 0$$

$$-2\bar{Y}_{st}\bar{Z}_{st}\left(-\frac{1}{2}\alpha_{1,h} g_{1,h}^2 \theta_{1,h}\rho_{x,z,h} C_{x,h} C_{z,h} + g_{1,h}\theta_{1,h}\rho_{y,z,h} C_{y,h} C_{z,h}\right) - 2\,\beta_{1,h}\bar{Z}_{st}^2 g_{1,h}^2 \theta_{1,h} C_{z,h}^2 = 0$$

$$\alpha_{1,h} = \frac{\left(\bar{Y}_{st}\rho_{x,z,h}C_{y,h} - \beta_{1,h}g_{1,h}\bar{Z}C_{,z,h}\right)}{\bar{Y}_{st}\rho_{x,z,h}C_{x,h}} \qquad (26b)$$

Equating (29a) to (29b), and solving for $\beta_{1,h}$ we have:

$$\beta_{1,h} = \frac{\bar{Y}_{st}C_{y,h}(\rho_{x,z,h}\rho_{y,x,h} - \rho_{y,z,h})}{\bar{Z}_{st}g_{1,h}C_{z,h}(\rho_{xz}^2 - 1)} \qquad (27a)$$

Substituting (30a) into (3.104a) and simplifying, we have

$$\alpha_{1,h} = \frac{2C_{y,h}(\rho_{x,z,h}\rho_{y,x,h} - \rho_{y,z,h})}{g_{1,h}C_{z,h}(\rho_{x,z,h}^2 - 1)} \qquad (27b)$$

Substituting (26a) and (27b) into (25), we have on simplification gives:

$$MSE(\bar{y}_{tv.st}) = \sum_{h=1}^{L} \frac{W_h^2 Y_{st}^2 \theta_{1,h} C_{y,h}^2}{\rho_{x,z,h}^2 - 1}\left(\rho_{y,z,h}^2 + \rho_{x,z,h}^2 + \rho_{y,x,h}^2 - 2\rho_{y,x,h}\rho_{x,z,h}\rho_{y,z,h} - 1\right)$$

Therefore,

$$MSE(\bar{y}_{tv.st}) = \bar{Y}_{st}^2 \sum_{h=1}^{L} W_h^2 \frac{C_{y,h}^2\theta_{1,h}}{\rho_{xzh}^2 - 1}\left(\rho_{y,z,h}^2 + \rho_{x,z,h}^2 + \rho_{y,x,h}^2 - 2\rho_{y,x,h}\rho_{x,z,h}\rho_{y,z,h} - 1\right)$$

where, $\rho_{y,x,h}, \rho_{x,z,h}, \rho_{y,z,h} \pm 1$.

Recall from (20) that:

$$Bias(\bar{y}_{tv.st}) = \bar{Y}_{st} \sum_{h=1}^{L} W_h^2 \left[-\frac{1}{2}\alpha_{1,h}g_{1,h}\theta_{1,h}\rho_{y,x,h}C_{x,h}C_{y,h} + \frac{1}{8}\alpha_{1,h}^2 g_{1,h}^2\theta_{1,h}C_{x,h}^2\right]$$

$$+\beta_{1,h}\bar{Z}_{st}\sum_{h=1}^{L} W_h^2\left[\frac{1}{2}\alpha_{1,h}g_{1,h}^2\theta_{1,h}\rho_{x,z,h}C_{x,h}C_{z,h}\right]$$

Substituting (30a) and (30b) into the above expression gives on simplification

$$Bias(\bar{y}_{tv.st}) = \frac{1}{2}\bar{Y}_{st}\sum_{h=1}^{L}\frac{W_h^2 C_{y,h}^2\theta_{1,h}}{\left(\rho_{x,z,h}^2 - 1\right)^2}\left(\rho_{x,z,h}\rho_{y,z,h} - \rho_{y,x,h}\right)^2;\ \rho_{x,z} \neq 1 \qquad (28)$$

***Corollary 1:***
The bias of the alternative hybrid estimator under stratified random sampling when the auxiliary variables are not transformed, $Bias(\bar{y}_{uv.st})$ is -1 times the bias of the alternative hybrid estimator when the auxiliary variables are transformed, $Bias(\bar{y}_{tv.st})$. i.e. $Bias(\bar{y}_{uv.st}) = -Bias(\bar{y}_{tv.st})$

***Corollary 2:***
The MSE of $\bar{y}_{tv.st}$ the alternative hybrid estimator under stratified random sampling when the auxiliary variables are transformed, is independent of g and equals the MSE of $\bar{y}_{uv.st}$, the alternative hybrid estimator when the auxiliary variables are not transformed,. i.e. $MSE(\bar{y}_{uv.st}) = MSE(\bar{y}_{tv.st})$

***Efficiency Criteria***
For two unbiased estimators $T$ and $T^*$ of $\theta$, we compare the variances of the estimatorsto determine their relative efficiency, i.e. Relative Efficiency of $T$ and $T^*$ is:

$$RE(T, T^*; \theta) = \frac{Var(T^*)}{Var(T)} \qquad (29)$$

We extend this to large sample variances of asymptotically unbiased estimators.
Let $\{T_i^*\}, i = 1, 2$ and $\{T_j\}, j = 1, 2, 3$ be asymptotically unbiased for $\tau(\theta)$, then it is said that $T_i^*$is Asymptotically EfficientRelative to$\{T_j\}$if:

$$ARE\left(T_j, T_i^*; \theta\right) = \lim_{n\to\infty}\frac{Var(T_i^*)}{Var(T_j)} \leq 1,\ \ \theta \in \Theta, i = 1, 2; j = 1, 2, 3 \qquad (30)$$

where $\{T_i^*\}, i = 1, 2$are the proposed and $\{T_j\}, j = 1, 2, 3$ and existing and competing estimators [9], [10], [11] under consideration in this study.

## Results

**Table 1: Statistics of Study Populations**

| Population Parameter | Population I | Population II | Population III |
|---|---|---|---|
| $N$ | 1000 | 1000 | 1000 |
| $\bar{Y}$ | 3000.32 | 126.72 | 125.69 |
| $\bar{X}$ | 2000.08 | 25.91 | 99.71 |
| $\bar{Z}$ | 1000.02 | 75.28 | 83.04 |
| $S_y^2$ | 97.50 | 5155.49 | 12839.33 |
| $S_x^2$ | 26.71 | 208.08 | 9597.71 |
| $S_z^2$ | 25.53 | 1832.56 | 6086.2 |
| $S_{yx}$ | −2.12 | −16.34 | −505.61 |
| $S_{yz}$ | 0.248 | 29.19 | −413.89 |
| $S_{xz}$ | −1.42 | −22.23 | −318.98 |
| $S_y$ | 9.87 | 71.81 | 113.31 |
| $S_x$ | 5.168 | 14.43 | 97.97 |
| $S_z$ | 5.052 | 42.81 | 78.01 |
| $C_y$ | 0.0033 | 0.718 | 0.902 |
| $C_x$ | 0.0026 | 0.557 | 0.98 |
| $C_z$ | 0.0051 | 0.569 | 0.94 |

**Table 2: Coefficient of Variation of Estimators for Populations I-III**

| | | Population I Sample size (n) | | | | Population II Sample size (n) | | | | Population III Sample size (n) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *CCP* | *Estimator* | 10 | 25 | 50 | 100 | 10 | 25 | 50 | 100 | 10 | 25 | 50 | 100 |
| $\rho_{HLL}$ | $\bar{y}_{uv.st}$ | 0.0116 | 0.0040 | 0.0057 | 0.0013 | 5.8650 | 2.5218 | 0.7995 | 0.2639 | 4.6854 | 0.8937 | 0.3714 | 0.1984 |
| | $\bar{y}_{tv.st}$ | 0.0116 | 0.0040 | 0.0057 | 0.0013 | 5.8148 | 2.4536 | 0.7990 | 0.2605 | 4.8743 | 3.6859 | 2.7917 | 1.4585 |
| | $\bar{y}_{Tet.st}$ | 0.0286 | 0.0258 | 0.0266 | 0.0108 | 10.7968 | 11.5826 | 5.7271 | 2.2202 | 17.8435 | 4.0003 | 2.7897 | 1.8392 |
| | $\bar{y}_{YT.st}$ | 0.0241 | 0.0229 | 0.0205 | 0.0080 | 7.4116 | 4.7111 | 2.5533 | 1.3140 | 8.2492 | 1.9296 | 1.2522 | 0.8143 |
| | $\bar{y}_{RP.st}$ | 0.0501 | 0.0441 | 0.0403 | 0.0113 | 17.8623 | 14.9265 | 7.1448 | 3.3589 | 23.7095 | 5.0702 | 3.3989 | 2.2009 |
| $\rho_{HHH}$ | $\bar{y}_{uv.st}$ | 0.0115 | 0.0034 | 0.003 | 0.0011 | 3.2046 | 2.4837 | 0.5505 | 0.2278 | 4.6751 | 0.7146 | 0.3265 | 0.2665 |
| | $\bar{y}_{tv.st}$ | 0.0115 | 0.0034 | 0.003 | 0.0011 | 3.2142 | 2.4249 | 0.5430 | 0.2269 | 4.4975 | 0.5274 | 0.2118 | 0.1777 |
| | $\bar{y}_{Tet.st}$ | 0.0493 | 0.0407 | 0.0278 | 0.0137 | 5.9747 | 6.2766 | 2.9863 | 1.3188 | 16.274 | 3.1830 | 2.2772 | 0.6634 |
| | $\bar{y}_{YT.st}$ | 0.0242 | 0.0225 | 0.0182 | 0.0087 | 7.3724 | 4.5912 | 2.4185 | 1.3007 | 8.2585 | 1.9117 | 1.2256 | 0.7242 |
| | $\bar{y}_{RP.st}$ | 0.0139 | 0.0216 | 0.0230 | 0.0021 | 15.3470 | 10.9000 | 5.1559 | 2.7904 | 17.316 | 3.1725 | 1.6658 | 1.4808 |
| $\rho_{LLL}$ | $\bar{y}_{uv.st}$ | 0.0211 | 0.0231 | 0.0179 | 0.0077 | 11.7215 | 7.4628 | 3.4823 | 2.3321 | 11.8764 | 2.5188 | 1.7998 | 1.3327 |
| | $\bar{y}_{tv.st}$ | 0.0211 | 0.0231 | 0.0179 | 0.0077 | 11.7988 | 7.4295 | 3.4880 | 2.3364 | 11.3808 | 3.5056 | 2.4046 | 1.2769 |
| | $\bar{y}_{Tet.st}$ | 0.0526 | 0.0436 | 0.0410 | 0.0178 | 14.0814 | 8.7589 | 3.3587 | 2.2388 | 17.5939 | 4.2813 | 2.6489 | 1.3338 |
| | $\bar{y}_{YT.st}$ | 0.0239 | 0.0232 | 0.0181 | 0.0082 | 6.8779 | 4.3559 | 1.9492 | 1.3424 | 8.24931 | 1.9331 | 1.2396 | 0.7157 |
| | $\bar{y}_{RP.st}$ | 0.0233 | 0.0269 | 0.0288 | 0.0123 | 20.5416 | 12.3171 | 4.7851 | 3.3885 | 20.0748 | 4.6881 | 2.8435 | 1.8062 |
| $\rho_{\bar{L}LL}$ | $\bar{y}_{uv.st}$ | 0.0238 | 0.0200 | 0.0172 | 0.0086 | 9.0483 | 7.4887 | 2.7368 | 1.3830 | 11.9018 | 2.2326 | 1.6046 | 1.2788 |
| | $\bar{y}_{tv.st}$ | 0.0238 | 0.0200 | 0.0172 | 0.0086 | 9.0073 | 7.5337 | 2.7484 | 1.3700 | 11.3827 | 1.4861 | 1.0958 | 1.2120 |
| | $\bar{y}_{Tet.st}$ | 0.0441 | 0.0319 | 0.0268 | 0.0120 | 12.3857 | 8.1705 | 3.6095 | 3.5320 | 17.2646 | 3.8966 | 2.521 | 1.5165 |
| | $\bar{y}_{YT.st}$ | 0.0358 | 0.0261 | 0.0204 | 0.0096 | 6.51863 | 4.3804 | 1.4923 | 0.8733 | 8.23653 | 1.915 | 1.222 | 0.7401 |
| | $\bar{y}_{RP.st}$ | 0.0532 | 0.0270 | 0.024 | 0.0108 | 14.7522 | 11.4140 | 4.3175 | 4.6453 | 20.0748 | 4.6881 | 2.8435 | 1.8062 |

**Table 3: Asymptotic Relative Efficiency of Estimators for Populations I-III**

| | | Population I Sample size (n) | | | | Population II Sample size (n) | | | | Population III Sample size (n) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CCP$ | $Estimator$ | 10 | 25 | 50 | 100 | 10 | 25 | 50 | 100 | 10 | 25 | 50 | 100 |
| $\rho_{HLL}$ | $\bar{y}_{uv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{tv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{Tet.st}$ | 0.1652 | 0.0238 | 0.0447 | 0.0142 | 0.2626 | 0.0514 | 0.0211 | 0.0133 | 0.1215 | 0.0418 | 0.0108 | 0.0073 |
| | $\bar{y}_{YT.st}$ | 0.2317 | 0.0304 | 0.0760 | 0.0261 | 0.6587 | 0.2957 | 0.1060 | 0.0407 | 0.3583 | 0.1448 | 0.0510 | 0.0363 |
| | $\bar{y}_{RP.st}$ | 0.0536 | 0.0081 | 0.0195 | 0.1283 | 0.1018 | 0.0327 | 0.0143 | 0.0061 | 0.1047 | 0.0250 | 0.0073 | 0.0051 |
| $\rho_{HHH}$ | $\bar{y}_{uv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{tv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{Tet.st}$ | 0.0546 | 0.0069 | 0.0116 | 0.0064 | 0.2593 | 0.1626 | 0.0353 | 0.0278 | 0.1215 | 0.0320 | 0.0092 | 0.0712 |
| | $\bar{y}_{YT.st}$ | 0.2254 | 0.0225 | 0.0272 | 0.0160 | 0.2013 | 0.2892 | 0.0536 | 0.0308 | 0.2974 | 0.0716 | 0.0301 | 0.0581 |
| | $\bar{y}_{RP.st}$ | 0.6803 | 0.0243 | 0.1093 | 0.2683 | 0.0417 | 0.0571 | 0.0125 | 0.0066 | 0.1633 | 0.0310 | 0.0171 | 0.0142 |
| $\rho_{LLL}$ | $\bar{y}_{uv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{tv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{Tet.st}$ | 0.1611 | 0.2780 | 0.1892 | 0.1858 | 0.6372 | 0.6603 | 1.1457 | 1.0326 | 0.6592 | 0.4186 | 0.4345 | 0.9644 |
| | $\bar{y}_{YT.st}$ | 0.7813 | 0.9851 | 0.9732 | 0.8874 | 3.1157 | 3.0239 | 3.0650 | 3.1501 | 1.8894 | 1.6563 | 1.8840 | 3.2561 |
| | $\bar{y}_{RP.st}$ | 0.8232 | 0.7260 | 0.3845 | 0.3918 | 0.4242 | 0.3539 | 0.5381 | 0.4764 | 0.5047 | 0.3360 | 0.3759 | 0.5227 |
| $\rho_{\bar{L}LL}$ | $\bar{y}_{uv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{tv.st}$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | $\bar{y}_{Tet.st}$ | 0.2912 | 0.3952 | 0.4081 | 0.5123 | 0.4750 | 0.8070 | 0.7525 | 0.2492 | 0.6844 | 0.5037 | 0.4796 | 0.7459 |
| | $\bar{y}_{YT.st}$ | 0.4420 | 0.5879 | 0.7067 | 0.7950 | 1.9992 | 2.9991 | 3.2556 | 2.6955 | 1.8949 | 1.6820 | 1.9388 | 3.0450 |
| | $\bar{y}_{RP.st}$ | 0.1998 | 0.5524 | 0.5107 | 0.6228 | 0.4637 | 0.4381 | 0.4096 | 0.2114 | 0.5046 | 0.3349 | 0.3759 | 0.5227 |

## Discussion of Results

In this paper, dominance is measured on the minimum Coefficient of Variation (CV) and Asymptotic Relative efficiency criteria defined by (30) above. From Table 2, we consider the first partition of the parameter space denoted by$\rho_{HLL}$. The proposed estimators have dominated other estimators in the entire subspace for populations I and II respectively. In case of Population III, one of the proposed estimators, $\bar{y}_{uv.st}$ dominated its variant, $\bar{y}_{tv.st}$ and other estimators in this particular parameter subspace ($\rho_{HLL}$). The proposed estimators have been shown to be asymptotically efficient when compared with the other estimators regarding population(I-III) as shown in Table 3. In the second partition of the parameter space defined by$\rho_{HHH}$, it is obviously noticed that the proposed estimators dominated the other estimators in the entire subspace for all the three populations (I, II, III) as depicted in the second compartment of Table 1. They are also asymptotically most efficient than the other estimators (Table 2). In case of the third subdivision of the parameter space defined by$\rho_{LLL}$,the proposed estimators dominated the other estimators for small sample sizes in Population I. However, as sample size increases the estimator $\bar{y}_{YT.st}$ becomes equally efficient as the proposed estimators. Interestingly, for populations II and III, it was observed that another estimator, $\bar{y}_{YT.st}$turns out to dominate the other estimators throughout the subdivision. So, it can be said that the proposed estimators are asymptotically more efficient than the others for Population I while the estimator, $\bar{y}_{YT.st}$ is asymptotically more efficient considering population I and II respectively. In the fourth parameter subinterval defined by $\rho_{\bar{L}LL}$,the proposed estimators dominated all the **o**ther estimators throughout the parameter space for population I. nevertheless, the estimator, $\bar{y}_{YT.st}$ dominated the proposed and the other competing estimators throughout the same subspace for population II and IIIrespectively (Table 2, last compartment). The proposed estimators are therefore, asymptotically more efficient when compared to other estimators for population I but otherwise in case of populations II and III

## Conclusion

In this paper, regression-cum-ratio exponential type estimators of finite population mean were proposed under stratified random sampling scheme. The expressions for the bias and Mean Square Error (MSE) of the estimators are derived and asymptotic properties of the proposed estimators investigated. A comprehensive simulation study to show the efficacy of the estimators as compared to conventional estimators was carried out using Coefficient of Variation as a performance measure. The results of the simulation study have shown that the proposed estimators are more efficient than most of the existing estimators considered in this study namely, [9] estimator [10] estimator, as well as [11] estimator in stratified random sampling.

**Declaration of conflicting interests**

The authors declared no potential conflict of interest

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