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Magnetohydrodynamic (MHD) of Blood Flow in a Stenosed Artery

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Abstract

This research modeled MHD third grade blood flow in a three layered stenosed artery. The most important rheological properties of blood, such as viscosity and the density have been modeled into the shear thinning/thickening parameters. Regular perturbation method was used to obtain the flow characteristics such as the flow velocity, the volume flow rate, the shear stress and the resistance to the flow. We considered a three layered stenosed artery of about 6unit length. We reduced the model into dimensionless parameters, hypothetical values were used. The obtained results showed that, the volume flow rate and the velocity increases with increase in the magnetic field intensity $H$and the shear thinning$\Lambda_1$and reduces with increase in the shear thickening Ω_1. The resistance increases with increase in the shear thickening Ω_1 and decrease with increase in magnetic field intensity H and the shear thinning Λ_1.

Keywords: Third grade fluid, Three Layered Flow, Magnetohydrodynamics (MHD), shear thinning/thickening, Stenosis

Introduction

Studies on blood flow have widely received attention in recent times due to its importance in human anatomy and physiology. It is a complex study because blood flow is a circulatory system where the flow is driven by the pumping action of the heart. Blood is pumped into arteries and transported into capillary beds where the exchange of gases and nutrients takes place and then transported back into the heart through veins. This circulating blood provides the nourishment (nutrients, oxygen, and other soluble factors) needed for supporting life [14].

Like the busy highways, blood vessels have to be well-constructed to withstand all the pressure that comes with blood circulation every minute of the day. To this end, the walls of the vessels are constructed in three layers known as tunics called tunica intima, tunica media and tunica external or adventitia respectively.

In real life, there are many materials that can be seen with characteristics of both elasticity and viscosity. These materials are referred to as non-Newtonian fluids which blood is one of such fluids. These fluids can only be described satisfactorily by the combination of both the theory of elasticity or viscosity. According to Buchanan *et al.* [1], blood behaves differently when flowing in large vessels, in which Newtonian behavior is expected and in medium and small vessels where non-Newtonian effects appear. Zeb *et al.* [16] studied steady flow of an incompressible, third-grade fluid in helical screw rheometer (HSR) by "unwrapping or flattening" the channel, lands, and the outside rotating barrel. Hayat *et al.* [5] considered steady boundary layer axisymmetric flow of third-grade fluid over a continuously stretching cylinder in the presence of magnetic field. They used homotopy analysis method (HAM) to solve the differential equations.

Several studies have been conducted on three layered fluid flow most of which centered on the flow of blood in a three layered stenosed artery. For example, Chaturani and Biswas [2] modelled Couette flow of blood as a three-layered flow. The model basically consists of a core (red-cell suspension) and plasma (a Newtonian fluid) in the top (near the moving plate) and bottom (near the stationary plate) layers. Flow is assumed to be steady and laminar and fluids are incompressible. Rekha and Usha [9] presented a three-layer model consisting of a core region of suspension of the erythrocytes in plasma (fluid) of viscosity and a peripheral layer of cell free plasma layer to represent blood flow in small capillary and compared with the two fluid model (Casson fluid model) and particle fluid mixture model of Srivastava. Dharmendra [3] constructed a mathematical model to examine the characteristics of three-layered blood flow through the oscillatory cylindrical tube (stenosed arteries). His analysis was restricted to propagation of small-amplitude harmonic waves, generated due to blood flow whose wave length is larger compared to the radius of the arterial segment. The impacts of viscosity of fluid in peripheral layer and intermediate layer on the interfaces, average flow rate, mechanical efficiency, trapping and reflux were discussed with the help of numerical and computational results. Pandey *et al.* [7] studied the theoretical study of two-dimensional peristaltic flow of power-law fluids in three layers with different viscosities. The analysis is carried out under low Reynolds number and long wavelength approximations.

In recent time, perturbation method has been applauded for solving non-linear models in fluid dynamics. Sankar and Hemlatha [12] Sankar and Lee [13] used the perturbation method to obtain the flow variables in their studies of the pulsatile flow of non –Newtonian fluid in stenosed artery by considering blood as Casson fluid and Herschel –Bulkley fluid with body acceleration. The method seemed consistent and Rekha and Usha [9] used the method in their three-layer fluid model and compared with the two-fluid model (Casson fluid model) and particle fluid mixture model of Srivastava and there were no known variations in the results. Sankar [11] used the perturbation method in his two-fluid model for pulsatile flow in catheterized blood vessels.

In all the above studies with a few on three layered fluid model, we are motivated from the above interesting models and results on blood flow through the stenosed arteries, to develop a mathematical model to study blood flow in three layers in presence of magnetic field also considering three different fluid flow models in the three layers. Perturbation method will be used to obtain approximate analytic solution to the model.

Formulation of the model

Consider a fully developed flow of blood axially symmetric, laminar, and pulsatile, in the axial direction through a circular tube with an axially symmetric mild stenosis which is influenced by magnetic field. It is assumed that the body fluid (blood) is flowing in three layers with the inner layer as a Casson fluid, the central layer of suspension of all erythrocytes as a third-grade fluid and the external layer of plasma as a Newtonian fluid. Consider blood as a magnetic fluid since red blood cells are a major bio-magnetic substance, blood flow will be influenced by the magnetic field. Hence in the present study, we considered the flow of blood to be unidirectional and in the axial direction as can be seen by the flow diagram below. Fernando [4], presented the Cauchy stress tensor for both Newtonian and non-Newtonian fluids by

$$\tau = -PI + \sum_{j=1}^{n} S_j \tag{1}$$

$S_j, j = 1,2,3$, are called the stress tensors, $P$is the pressure force due to fluid flow. For the third-grade fluid we have $n = 3$ and the first three tensors S_j are given by

$$S_1 = \overline{\mu} A_1 \tag{2}$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2 \tag{3}$$

$$S_3 = \beta_1 A_3 + \beta_2 (A_2 A_1 + A_1 A_2) + \beta_3 (tr A_1^2) A_1 \tag{4}$$

Where $\overline{\mu}$ is the coefficient of sheer viscosity and $\alpha_i, (i = 1,2), \beta_i, (i = 1,2,3)$ are material constants. A_n Are called RivlinEricksen tensors and are defined by the recursion relation

$$A_n = \frac{D}{Dt} A_{n-1} + A_{n-1}(\nabla \overline{u}) + (\nabla \overline{u})^T A_{n-1}, n > 1 \tag{5}$$

$$A_1 = (\nabla \overline{u}) + (\nabla \overline{u})^T \tag{6}$$

When$\beta_j = 0, (j = 1,2,3)$, then, the above model reduces to second grade fluid model and if $\alpha_i = 0, (i = 1,2)$and $\beta_j = 0, (j = 1,2,3)$, the model reduces to classical Navier stokes viscous fluid model (Fernando, 2008).

Let the velocity field for the fluid flow be given as a vector field, we assume the flow to be in the positive z-direction. This means that the pressure gradient must be negative and the following must hold.

i). The velocity field is not dependent on the coordinate z and θ. That is, $\overline{u}_{rr}, \overline{u}_{\theta\theta}, \overline{u}_{zz}, \overline{u}_{r\theta}, \overline{u}_{rz}, \overline{u}_{\theta r}, \overline{u}_{\theta z}, \overline{u}_{z\theta} = 0$

ii). The extra stress are also not dependent on the coordinate z and θ. That is, $\tau_{rr}, \tau_{\theta\theta}, \tau_{zz}, \tau_{r\theta}, \tau_{\theta r} = \tau_{\theta z} = 0$

iii). The sheer stress $\tau_{\theta z}$ and $\tau_{z\theta}$ on planes through the axis of the pipe are zero due to symmetry.

This implies that the velocity field and the shear stresses are functions of r alone. Hence, the constitutive equation of motion for a third-grade fluid flow is

$$\tau = \tau_{zr} = \overline{\mu} \frac{\partial \overline{u}}{\partial \overline{r}} + \alpha_1 \frac{\partial^2 \overline{u}}{\partial \overline{t} \partial \overline{r}} + 2\beta_3 \left(\frac{\partial \overline{u}}{\partial \overline{r}} \right)^3 \tag{7}$$

According to Sandoo *et al*, (2015)[10], the tunica media contains a large amount of smooth muscles that allows a more efficient exchange of gases and nutrients in blood within the capillary beds. Thus, due to the presence of hemoglobin (iron compound) in the red blood cells in that layer, we regard blood in the layer as a suspension of magnetic particles. Hence, the continuity equation and the momentum equations are respectively given by

$$div \overline{u} = 0 \tag{8}$$

$$\overline{\rho}_c \frac{\partial}{\partial \overline{t}} \overline{u}_c = -\frac{\partial}{\partial \overline{z}} \overline{P} - \frac{1}{r} \frac{\partial}{\partial \overline{r}} (\overline{r \tau_c}) \tag{9}$$

$$\overline{\rho}_T \frac{\partial}{\partial \overline{t}} \overline{u}_T = -\frac{\partial}{\partial \overline{z}} \overline{P} - \frac{1}{r} \frac{\partial}{\partial \overline{r}} (\overline{r \tau_T}) + \overline{\mu}_0 M \frac{\partial}{\partial \overline{z}} \overline{H} \tag{10}$$

$$\overline{\rho}_N \frac{\partial}{\partial \overline{t}} \overline{u}_N = -\frac{\partial}{\partial \overline{z}} \overline{P} - \frac{1}{r} \frac{\partial}{\partial \overline{r}} (\overline{r \tau_N}) \tag{11}$$

The pressure gradient is pulse; it is a function of the direction of flow $\overline{z}$and time $\overline{t}$, we assume

$$-\frac{\partial}{\partial \overline{z}} \overline{p} = \overline{q_0} + A_1 \cos \overline{\omega} \, \overline{t}, \quad \overline{t} \geq 0 \tag{12}$$

The relations between the shear stress and the strain rate of the fluid in motion in the three layers, that is, the constitutive equation of motion of the fluid flow are given by

$$\left.\begin{array}{c}\sqrt{\overline{\tau}_c} = \sqrt{\overline{\tau}_y} + \sqrt{-\overline{\mu}\frac{\partial}{\partial \overline{r}}\overline{u}_c}\, , if\ \sqrt{\overline{\tau}_c} \geq \sqrt{\overline{\tau}_y} \\ \overline{R}_p(\overline{z}) \leq \overline{r} \leq \overline{R}_2(\overline{z})\end{array}\right\} \quad (13)$$

$$\left.\begin{array}{c}\overline{\tau}_T = -\overline{\mu}_T\frac{\partial}{\partial \overline{r}}\overline{u}_T - \alpha_1\frac{\partial^2}{\partial \overline{r}\partial \overline{t}}\overline{u}_T - 2\beta_3\left(\frac{\partial}{\partial \overline{r}}\overline{u}_T\right)^3 \\ if\ \ \overline{R}_2(\overline{z}) \leq \overline{r} \leq \overline{R}_1(\overline{z})\end{array}\right\} \quad (14)$$

$$\left.\begin{array}{c}\overline{\tau}_N = -\overline{\mu}\frac{\partial}{\partial \overline{r}}\overline{u}_N \\ if\ \ \overline{R}_1(\overline{z}) \leq \overline{r} \leq \overline{R}(\overline{z})\end{array}\right\} \quad (15)$$

The geometry of the stenosis is given in figure1

$$\left.\begin{array}{c}\frac{\partial}{\partial \overline{r}}\overline{u} = 0 \\ if\ \ 0 \leq \overline{r} \leq \overline{R}(\overline{z})\end{array}\right\} \quad (16)$$

The boundary conditions are given as follows.

$\overline{\tau}_C$is finite at $\overline{r} = 0$ (17)

$\overline{u}_N = 0$ at $\overline{r} = \overline{R}(\overline{z})$ (18)

$\overline{\tau}_C = \overline{\tau}_T$ and $\overline{u}_C = \overline{u}_T$ at $\overline{r} = \overline{R}_2(\overline{z})$ (19)

$\overline{\tau}_N = \overline{\tau}_T$ and $\overline{u}_N = \overline{u}_T$ at $\overline{r} = \overline{R}_1(\overline{z})$ (20)

$\overline{u}_C = 0$ at $\overline{r} = 0$ (21)

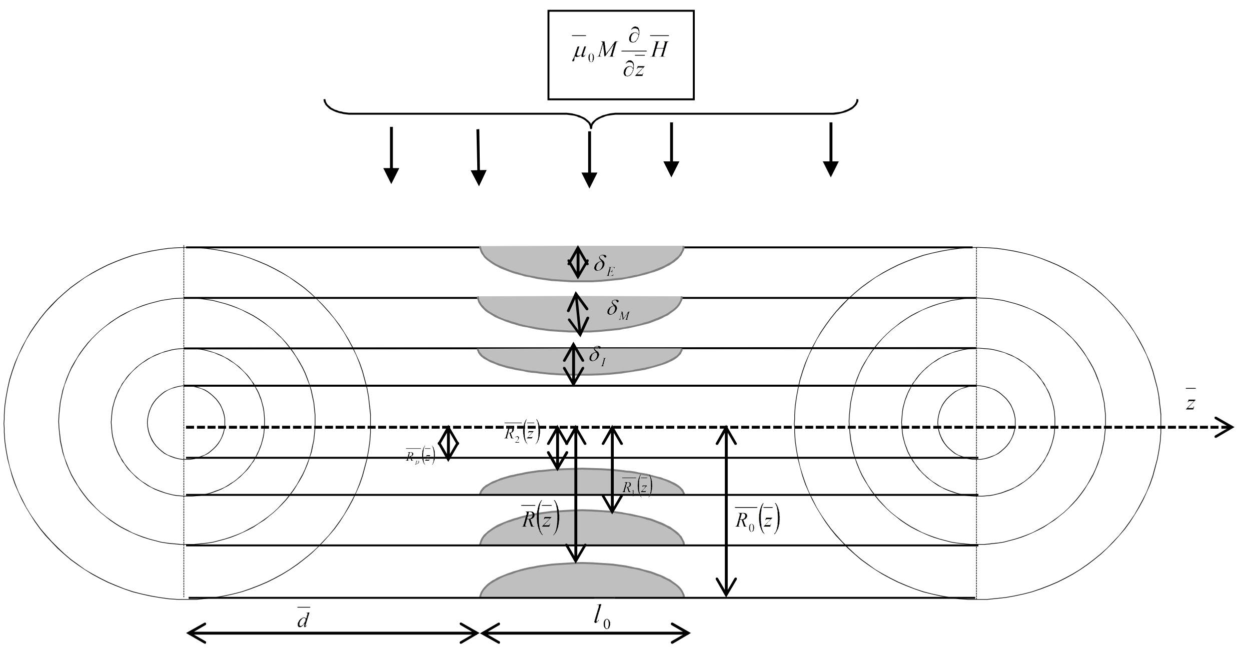

Figure 1: Geometry of the three layered stenosed artery

and is defined by

$$\overline{R}(\overline{z}) = \begin{cases}\begin{cases}\overline{R_0} - \left(\frac{\delta_E}{2}\right)\left\{1 + cos\left[\frac{2\pi}{\overline{l_0}}\left(\overline{z} - \overline{d} - \frac{\overline{l_0}}{2}\right)\right]\right\} \\ \quad in\ \ \overline{d} \leq \overline{z} \leq \overline{d} + \overline{l_0}\end{cases} \\ \overline{R_0} \text{ in the normal artery region}\end{cases} \quad (22)$$

$$\overline{R_1}(\overline{z}) = \begin{cases}\begin{cases}\beta\overline{R_0} - \left(\frac{\delta_M}{2}\right)\left\{1 + cos\left[\frac{2\pi}{\overline{l_0}}\left(\overline{z} - \overline{d} - \frac{\overline{l_0}}{2}\right)\right]\right\} \\ \quad in\ \overline{d} \leq \overline{z} \leq \overline{d} + \overline{l_0}\end{cases} \\ \beta\overline{R_0} \text{ in the normal artery region}\end{cases} \quad (23)$$

$$\overline{R_2}(\overline{z}) = \begin{cases} \left\{ \beta_1\overline{R_0} - \left(\frac{\overline{\delta_I}}{2}\right)\left\{1 + \cos\left[\frac{2\pi}{\overline{l_0}}\left(\overline{z} - \overline{d} - \frac{\overline{l_0}}{2}\right)\right]\right\} \right. \\ \quad in\ \overline{d} \le \overline{z} \le \overline{d} + \overline{l_0} \\ \beta_1\overline{R_0} \text{ in the normal artery region} \end{cases} \quad (24)$$

Where β is the ratio of the tunica media radius to the normal artery radius. β_1 is the ratio of the tunica intima radius to the normal artery radius $\overline{\delta_E}, \overline{\delta_M}$ and $\overline{\delta_I}$ are respectively the height of the stenosis in the tunics.

We introduce the following non-dimensional variables to reduce our model into a model with dimensionless variables for easy computation.

$$z = \frac{\overline{z}}{\overline{R_0}}, r = \frac{\overline{r}}{\overline{R_0}}, z = \frac{\overline{z}}{\overline{R_0}}, R(z) = \frac{\overline{R}(\overline{z})}{\overline{R_0}}, R_1(z) = \frac{\overline{R_1}(\overline{z})}{\overline{R_0}}, q(z) = \frac{\overline{q}(\overline{z})}{\overline{R_0}}, \delta_E = \frac{\overline{\delta_E}}{\overline{R_0}}, \delta_M = \frac{\overline{\delta_M}}{\overline{R_0}}, \delta_I = \frac{\overline{\delta_I}}{\overline{R_0}}, t = \overline{\omega}\overline{t}, \alpha_C^2 = \frac{\overline{R_0^2}\overline{\omega\rho_C}}{\overline{\mu_C}}, \alpha_T^2 = \frac{\overline{R_0^2}\overline{\omega\rho_T}}{\overline{\mu_T}}, \alpha_N^2 = \frac{\overline{R_0^2}\overline{\omega\rho_N}}{\overline{\mu_N}}, \ell = \frac{\overline{A_1}}{\overline{q_0}}, u_C = \frac{\overline{u_C}}{\frac{\overline{q_0}\overline{R_0^2}}{4\overline{\mu_C}}}, u_T = \frac{\overline{u_T}}{\frac{\overline{q_0}\overline{R_0^2}}{4\overline{\mu_T}}}, H = \frac{\overline{H}}{\overline{q_0}}, \tau_c = \frac{\overline{\tau_c}}{\frac{\overline{q_0}\overline{R_0}}{2}}, \tau_T = \frac{\overline{\tau_T}}{\overline{q_0}\overline{R_0}}, \tau_N = \frac{\overline{\tau_N}}{\overline{q_0}\overline{R_0}/2}, \theta = \frac{\overline{\tau_y}}{\overline{q_0}\overline{R_0}/2}, \beta = \frac{\overline{a_0}^2}{\overline{q_0}}. \quad (25)$$

Thus, substituting (25) into (8) to (24) gives

$$\frac{\alpha_c^2}{4}\frac{\partial}{\partial t}u_c = (1 + \ell\cos(t)) - \frac{1}{2r}\frac{\partial}{\partial r}(r\tau_c) \quad (26)$$

$$\frac{\alpha_T^2}{4}\frac{\partial}{\partial t}u_T = (1 + \ell\cos(t)) - \frac{1}{2r}\frac{\partial}{\partial r}(r\tau_T) + F\frac{\partial}{\partial z}H \ (27) \quad (27)$$

$$\frac{\alpha_N^2}{4}\frac{\partial}{\partial t}u_N = (1 + \ell\cos(t)) - \frac{1}{2r}\frac{\partial}{\partial r}(r\tau_N) \quad (28)$$

and the constitutive equations becomes

$$\left.\begin{array}{c} -\frac{\partial}{\partial r}u_c = 2\left(\tau_c - 2\sqrt{\theta\tau_c} + \theta\right), \\ if\ \sqrt{\tau_c} \ge \sqrt{\theta} \\ and \\ R_p(z) \le r \le R_2(z) \end{array}\right\} \quad (29)$$

If we let $\frac{\alpha_1\overline{\omega}}{\overline{\mu_T}} = \Omega, \frac{\beta_3\overline{q_0^2R_0^2}}{\overline{\mu_T^3}} = \Lambda$

and further that $\Lambda = 16\alpha^2\Lambda_1$ and $\Omega = 2\alpha^2\Omega_1$

$$\left.\begin{array}{c} \tau_T = -\frac{1}{2}\frac{\partial u_T}{\partial r} - \alpha^2\Omega_1 - \alpha^2\Lambda_1\left(\frac{\partial u_T}{\partial r}\right)^3, \\ if\ \ R_2(z) \le r \le R_1(z) \end{array}\right\} \quad (30)$$

$$\left.\begin{array}{c} \tau_N = -\frac{1}{2}\frac{\partial}{\partial r}u_N \\ if\ R_1(z) \le r \le R(z) \end{array}\right\} \quad (31)$$

τ_C is finite at $r = 0$ (32)

$u_N = 0$ at $r = R(z)$ (33)

$\tau_C = \tau_T$ and $u_T = u_C$ at $r = R_2(z)$ (34)

$\tau_T = \tau_N$ and $u_T = u_N$ at $r = R_1(z)$ (35)

$u_C = 0$ at $r = 0$ (36)

$$R(z) = \begin{cases} \left\{1 - \left(\frac{\delta_N}{2}\right)\left\{1 + \cos\left[\frac{2\pi}{l_0}\left(z - d - \frac{l_0}{2}\right)\right]\right\}\right. \\ \quad in\ d \le z \le d + l_0 \\ 1, \quad \text{in the normal artery region.} \end{cases} \quad (37)$$

$$R_1(z) = \begin{cases} \left\{\beta - \left(\frac{\delta_T}{2}\right)\left\{1 + \cos\left[\frac{2\pi}{l_0}\left(z - d - \frac{l_0}{2}\right)\right]\right\}\right. \\ \quad in\ \ d \le z \le d + l_0 \\ \beta \quad \text{in the normal artery region} \end{cases} \quad (38)$$

and

$$R_2(z) = \begin{cases} \left\{\beta_1 - \left(\frac{\delta_c}{2}\right)\left\{1 + \cos\left[\frac{2\pi}{l_0}\left(z - d - \frac{l_0}{2}\right)\right]\right\}\right. \\ \quad in\ d \le z \le d + l_0 \\ \beta_1 \quad \text{in the normal artery region} \end{cases} \quad (39)$$

Solution of the Three-Layered Model Equations using Regular Perturbation Method

We present the following perturbation series as thus,

$$\left.\begin{array}{l} \tau_N = \tau_{N0} + \alpha_N^2\tau_{N1} + \cdots \\ \tau_T = \tau_{T0} + \alpha_T^2\tau_{T1} + \cdots \\ \tau_C = \tau_{C0} + \alpha_C^2\tau_{C1} + \cdots \\ u_N = u_{N0} + \alpha_N^2u_{N1} + \cdots \\ u_T = u_{T0} + \alpha_T^2u_{T1} + \cdots \\ u_C = u_{C0} + \alpha_C^2u_{C1} + \ldots \end{array}\right\} \quad (40)$$

Let us expand the flow characteristics of (26) to (31) in the perturbation series of α^2 and equate coefficients of α as

α^0:

$$\begin{cases} 4(1+\ell\cos(t)) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{C0}) = 0 \\ 4(1+\ell\cos(t)) + F\frac{\partial}{\partial z}H - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{T0}) = 0 \\ 4(1+\ell\cos(t)) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{N0}) = 0 \\ -\frac{\partial}{\partial r}(u_{C0}) = 2\left(\tau_{C0} - \sqrt{(\theta\tau_{C0})} + \theta\right) \\ \tau_{T0} = \frac{1}{2}\frac{\partial}{\partial r}(u_{T0}) \\ \tau_{N0} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{N0}) \end{cases} \quad (41)$$

$$\alpha^2: \begin{cases} \frac{\partial}{\partial t}(u_{C0}) = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{C1}) \\ \frac{\partial}{\partial t}(u_{T0}) = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{T1}) \\ \frac{\partial}{\partial t}(u_{N0}) = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{N1}) \\ -\frac{\partial}{\partial r}(u_{C1}) = 2\tau_{C1}\left(1 - \sqrt{\left(\frac{\theta}{\tau_{C0}}\right)}\right) \\ \tau_{T1} = \frac{1}{2}\frac{\partial}{\partial r}(u_{T1}) + \Omega_1\frac{\partial^2}{\partial t\partial r}(u_{T0}) \\ \qquad + \Lambda_1\left(\frac{\partial}{\partial r}(u_{T0})\right)^3 \\ \tau_{N1} = -\frac{1}{2}\frac{\partial}{\partial r}(u_{N1}) \end{cases} \quad 42$$

$$\tau_{C0} = A(r) \quad (43)$$

$$\tau_{T0} = A(r) + \frac{1}{4}F\frac{\partial}{\partial z}H\left(r - \frac{R_2^2}{r}\right) \quad (44)$$

$$\tau_{N0} = A(r) + \frac{1}{4}F\frac{\partial}{\partial z}H\left(\frac{R_1^2 - R_2^2}{r}\right) \quad (45)$$

$$u_{N0} = A(R^2 - r^2) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_1^2 - R_2^2)\,ln\left(\frac{R}{r}\right) \quad (46)$$

$$u_{C0} = 2\theta(R_2 - r) + A(R^2 - 2R_1^2 + 2R_2^2 - r^2) + \frac{1}{4}F\frac{\partial}{\partial z}H(R_2^2 - R_1^2) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_1^2 - R_2^2)\,ln\left(\frac{R}{R_1}\right) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_2^2)\,ln\left(\frac{R_1}{R_2}\right) + \frac{4}{3}\sqrt{\theta A}\left(r^{\frac{3}{2}} - R_2^{\frac{3}{2}}\right) \quad (47)$$

$$\tau_{C1} = \frac{1}{8}B(2R^2r - 4R_1^2r + 4R_2^2r - r^3) + \frac{1}{42}C\left(4r^{\frac{5}{2}} - 7R_2^{\frac{3}{2}}r\right) \quad (48)$$

$$\tau_{T1} = \frac{1}{8}B(2R^2r - 4R_1^2r + r^3) + \frac{1}{4}B\left(\frac{R_2^4}{r}\right) - \frac{1}{14}C\left(\frac{R_2^{\frac{7}{2}}}{r}\right) \quad (49)$$

$$\tau_{N1} = \frac{1}{8}B(2R^2r - r^3) + \frac{1}{4}B\left(\frac{R_2^4 - R_1^4}{r}\right) - \frac{1}{14}C\left(\frac{R_2^{\frac{7}{2}}}{r}\right) \quad (50)$$

$$u_{N1} = \frac{1}{16}B(3R^4 - 4R^2r^2 + r^4) + \frac{1}{2}B(R_2^4 - R_1^4)\,ln\left(\frac{R}{r}\right) + \frac{1}{7}C\left(R_2^{\frac{7}{2}}\right)ln\left(\frac{r}{R}\right) \quad (51)$$

$$u_{T1} = \Omega_1 B(r^2 - R_1^2) + \frac{3}{4}\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3(R_2^4)\,ln\left(\frac{r}{R_1}\right) + \frac{1}{7}C\left(R_2^{\frac{7}{2}}\right)ln\left(\frac{R_1}{r}\right) + \frac{1}{2}B(R_2^4 - R_1^4)\,ln\left(\frac{R}{R_1}\right) + \frac{1}{16}B(3R^4 + 4R^2r^2 - 8R_1^2r^2 + r^4 - 8R^2R_1^2 + 8R_1^4) + \frac{1}{2}B(R_2^4)\,ln\left(\frac{r}{R_1}\right) + \frac{1}{7}C\left(R_2^{\frac{7}{2}}\right)ln\left(\frac{R_1}{R}\right) + 3\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_2^4)\,ln\left(\frac{R_1}{r}\right) + 4\Lambda_1 A^3(R_1^4 - r^4) + 3\Lambda_1 A^2\left(F\frac{\partial}{\partial z}H\right)(R_1^4 - 2R_2^2R_1^2 - r^4 + 2R_2^2r^2) + \frac{3}{4}\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_1^4 - 4R_2^2R_1^2 - r^4 + 4R_2^2r^2) + \frac{1}{16}\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - 6R_2^2R_1^2 - r^4 + 6R_2^2r^2 + \frac{2R_2^6}{R_1^2} - \frac{2R_2^6}{r^2}\right) \quad (52)$$

$$u_{C1} = -\frac{1}{16}B(4R^2r^2 - 8R_1^2r^2 + 8R_2^2r^2 - r^4) - \frac{1}{63}E\left(10R_2^3 + 4r^3 - 14R_2^{\frac{3}{2}}r^{\frac{3}{2}}\right) + \Omega_1 B(R_2^2 - R_1^2) + \frac{1}{84}F\left(28R^2r^{\frac{3}{2}} - 56R_1^2r^{\frac{3}{2}} - 6r^{\frac{7}{2}} - 28R^2R_2^{\frac{3}{2}} + 56R_1^2R_2^{\frac{3}{2}} + 6R_2^{\frac{7}{2}}\right) + \frac{3}{4}\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3(R_2^4)\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{294}C\left(33R_2^{\frac{7}{2}} + 16r^{\frac{7}{2}} - 49R_2^{\frac{3}{2}}r^2\right) + 4\Lambda_1(1+\ell\cos(t))^3(R_1^4 - R_2^4) + \frac{1}{7}C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{R}\right) + 3\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_2^4)\ln\left(\frac{R_1}{R_2}\right) + \frac{1}{7}C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{R_2}\right) + \frac{1}{16}\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - 6R_2^2R_1^2 + 3R_2^4 + \frac{2R_2^6}{R_1^2}\right) + \frac{1}{16}B(3R^4 + 8R^2R_2^2 - 16R_1^2R_2^2 + 8R_2^4 - 8R^2R_1^2 + 8R_1^4) + \frac{1}{2}B(R_2^4 - R_1^4)\ln\left(\frac{R}{R_1}\right) + \frac{1}{2}B(R_2^4)\ln\left(\frac{R_2}{R_1}\right) + 3\Lambda_1 A^2\left(F\frac{\partial}{\partial z}H\right)(R_1^4 - 2R_2^2R_1^2 + R_2^4) + \frac{3}{4}\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_1^4 - 4R_2^2R_1^2 + 3R_2^4) \quad (53)$$

If we let

$A = (1+\ell\cos(t))$, $B = \ell\sin(t)$, $C = \frac{\theta\ell\sin(t)}{\sqrt{\theta(1+\ell\cos(t))}}$,

$E = \frac{\sqrt{\theta^3}\ell\sin(t)}{(1+\ell\cos(t))}$, $F = \ell\sin(t)\sqrt{\frac{\theta}{(1+\ell\cos(t))}}$.

The solution of the system (41) and (42) together with the boundary conditions gives

$$\tau_C = A(r) + \frac{1}{8}\alpha_C^2 B(2R^2r - 4R_1^2r + 4R_2^2r - r^3) + \frac{1}{42}\alpha_C^2 C\left(4r^{\frac{5}{2}} - 7R_2^{\frac{3}{2}}r\right) \quad (54)$$

$$\tau_T = A(r) + \frac{1}{2}F\frac{\partial}{\partial z}H\left(\frac{R_1^2 - R_2^2}{r}\right) - \frac{1}{14}\alpha_T^2 C\left(\frac{R_2^{\frac{7}{2}}}{r}\right) + \frac{1}{8}\alpha_T^2 B(2R^2r - 4R_1^2r + r^3)\frac{1}{4}\alpha_T^2 B\left(\frac{R_2^4}{r}\right) \quad (55)$$

$$u_T = A(R^2 - 2R_1^2 + r^2) + \frac{1}{2}F\frac{\partial}{\partial z}H(r^2 - R_1^2) + F\frac{\partial}{\partial z}H(R_1^2 - R_2^2)\ln\left(\frac{R}{R_1}\right) + 4\alpha_T^2\Lambda_1 A^3(R_1^4 - r^4) + F\frac{\partial}{\partial z}H(R_2^2)\ln\left(\frac{R_1}{r}\right) + \frac{3}{4}\alpha_T^2\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_1^4 - 4R_2^2R_1^2 - r^4 + 4R_2^2r^2) + \frac{1}{2}\alpha_T^2 B(R_2^4 - R_1^4)\ln\left(\frac{R}{R_1}\right) + \frac{1}{2}\alpha_T^2 B(R_2^4)\ln\left(\frac{r}{R_1}\right) + 3\alpha_T^2\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_2^4)\ln\left(\frac{R_1}{r}\right) + \frac{1}{16}\alpha_T^2 B(3R^4) + \frac{1}{16}\alpha_T^2 B(4R^2r^2 - 8R_1^2r^2 + r^4 - 8R^2R_1^2 + 8R_1^4) + \frac{1}{7}\alpha_T^2 C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{R}\right) + \frac{1}{7}\alpha_T^2 C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{r}\right) + 3\alpha_T^2\Lambda_1 A^2\left(F\frac{\partial}{\partial z}H\right)(R_1^4 - 2R_2^2R_1^2 - r^4 + 2R_2^2r^2) + \Omega_1\alpha_T^2 B(r^2 - R_1^2) + \frac{3}{4}\alpha_T^2\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3(R_2^4)\ln\left(\frac{r}{R_1}\right) + \frac{1}{16}\alpha_T^2\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - 6R_2^2R_1^2 - r^4 + 6R_2^2r^2 + \frac{2R_2^6}{R_1^2} - \frac{2R_2^6}{r^2}\right) \quad (56)$$

$$\tau_N = A(r) + \frac{1}{2}F\frac{\partial}{\partial z}H\left(\frac{R_1^2 - R_2^2}{r}\right) - \frac{1}{14}\alpha_N^2 C\left(\frac{R_2^{\frac{7}{2}}}{r}\right) + \frac{1}{4}\alpha_N^2 B\left(\frac{R_2^4 - R_1^4}{r}\right) + \frac{1}{8}\alpha_N^2 B(2R^2r - r^3) \quad (57)$$

$$u_N = A(R^2 - r^2) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_1^2 - R_2^2)\ln\left(\frac{R}{r}\right) + \frac{1}{16}\alpha_N^2 B(3R^4 - 4R^2r^2 + r^4) + \frac{1}{2}\alpha_N^2 B(R_2^4 - R_1^4)\ln\left(\frac{R}{r}\right) + \frac{1}{7}\alpha_N^2 C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{r}{R}\right) \quad (58)$$

$$u_C = 2\theta(R_2 - r) + A(R^2 - 2R_1^2 + 2R_2^2 - r^2) + \frac{1}{4}F\frac{\partial}{\partial z}H(R_2^2 - R_1^2) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_1^2 - R_2^2)\ln\left(\frac{R}{R_1}\right) + \frac{1}{2}F\frac{\partial}{\partial z}H(R_2^2)\ln\left(\frac{R_1}{R_2}\right) + \frac{1}{7}\alpha_C^2 C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{R}\right) + \frac{1}{16}\alpha_C^2\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3\left(R_1^4 - 6R_2^2R_1^2 + 3R_2^4 + \frac{2R_2^6}{R_1^2}\right) + 3\alpha_C^2\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_2^4)\ln\left(\frac{R_1}{R_2}\right) + \frac{1}{294}\alpha_C^2 C\left(33R_2^{\frac{7}{2}} + 16r^{\frac{7}{2}} - 49R_2^{\frac{3}{2}}r^2\right) + \Omega_1\alpha_C^2 B(R_2^2 - R_1^2) + \frac{4}{3}\sqrt{\theta A}\left(r^{\frac{3}{2}} - R_2^{\frac{3}{2}}\right) - \frac{1}{16}\alpha_C^2 B(4R^2r^2 - 8R_1^2r^2 + 8R_2^2r^2 - r^4) - \frac{1}{63}\alpha_C^2 E\left(10R_2^3 + 4r^3 - 14R_2^{\frac{3}{2}}r^{\frac{3}{2}}\right) + \frac{1}{2}\alpha_C^2 B(R_2^4 - R_1^4)\ln\left(\frac{R}{R_1}\right) + \frac{1}{7}\alpha_C^2 C\left(R_2^{\frac{7}{2}}\right)\ln\left(\frac{R_1}{R_2}\right) + \frac{1}{84}\alpha_C^2 F\left(28R^2r^{\frac{3}{2}} - 56R_1^2r^{\frac{3}{2}} - 6r^{\frac{7}{2}} - 28R^2R_2^{\frac{3}{2}} + 56R_1^2R_2^{\frac{3}{2}} + 6R_2^{\frac{7}{2}}\right) + \frac{3}{4}\alpha_C^2\Lambda_1\left(F\frac{\partial}{\partial z}H\right)^3(R_2^4)\ln\left(\frac{R_2}{R_1}\right) + 3\alpha_C^2\Lambda_1 A^2\left(F\frac{\partial}{\partial z}H\right)(R_1^4 - 2R_2^2R_1^2 + R_2^4) + \frac{3}{4}\alpha_C^2\Lambda_1 A\left(F\frac{\partial}{\partial z}H\right)^2(R_1^4 - 4R_2^2R_1^2 + 3R_2^4) + \frac{1}{2}\alpha_C^2 B(R_2^4)\ln\left(\frac{R_2}{R_1}\right) + 4\alpha_C^2\Lambda_1 A^3(R_1^4 - R_2^4) + \frac{1}{16}\alpha_C^2 B(3R^4 + 8R^2R_2^2 - 16R_1^2R_2^2 + 8R_2^4 - 8R^2R_1^2 + 8R_1^4) \quad (59)$$

The volume flow rate is given by

$$Q_{three} = 2\pi\left[\int_0^{R_2(z)} u_C(r,t)rdr + \int_{R_2(z)}^{R_1(z)} u_T(r,t)rdr + \int_{R_1(z)}^{R(z)} u_N(r,t)rdr\right] \quad (60)$$

The resistance to the fluid flow in the three layers is given by

$$\lambda_{three} = \frac{-\frac{\partial}{\partial z}(P)}{Q_{three}} \quad (61)$$

$$\lambda_{three} = \frac{A}{Q_{three}} \quad (62)$$

The wall shear stress of the three-layered fluid flow is given by

$$\tau 3_w = \tau_N/_{r=R} \quad (63)$$

$$\tau 3_w = A(R) + \frac{1}{2}F\frac{\partial}{\partial z}H\left(\frac{R_1^2 - R_2^2}{R}\right) + \frac{1}{8}\alpha_N^2 B(R^3) - \frac{1}{14}\alpha_N^2 C\left(\frac{R_2^{\frac{7}{2}}}{R}\right) + \frac{1}{4}\alpha_N^2 B\left(\frac{R_2^4 - R_1^4}{R}\right) \quad (64)$$

Results and Discussion

Blood as a third-grade fluid possesses two properties, the shear thickening and shear thinning properties which are vital to this study. Increasing or decreasing these properties will affect the flow rate positively or negatively as the case may be. In parametric research, numerical simulations are very useful tools. With numerical experiments, it is possible to extract information difficult or impossible to obtain in the laboratory, in most cases, giving a better understanding of the physics of the problem under study. To this end, we have used the following to simulate the model; The dimensionless amplitude$\ell = 0.2, d = 2, l = 1$,. The Womersley numbers $\alpha_C^2 = 1.1$, $\alpha_T^2 = 1.2$and $\alpha_N^2 = 1.3$are used respectively. The Womersley number is the ratio of unsteady inertial forces to viscous forces in the flow. It ranges from as large as about 20 in the aorta, significantly greater than 1 in all large arteries, to as small as 10^{-3} in the capillaries [8]. This put our decision for the choice of the Womersley numbers as reasonable. The value of β is taken as 0.8, β_1 as 0.6 respectively. The value 0.18 is used for δ_N, 0.17 for δ_T and 0.16 for δ_C.The model parameters were reduced to dimensionless parameters and hypothetical parameter values are thought of as a good guide. We used maple computer software to numerically simulate the model.

According to Ku [6], Blood flow and pressure are unsteady and the cyclic nature of the heart pump creates pulsatile conditions in all arteries. The heart pumps blood in alternating cycles called *systole* and *diastole*. Blood volume flow rate occasioned by unsteady Pressure have characteristic pulsatile shapes that vary in different parts of the arterial system, as illustrated by Figures 2,3,4,5,6 and 7. Blood can be regarded as magnetic fluid, in which red blood cells are magnetic in nature. Liquid carriers in the blood contain the magnetic suspension of the particle [15]. Figures 2and 3 shows an increase in the volume flow rate as the applied magnetic field intensity increases. The flow rate begins to reduce to a low flow rate over time (Figure 8). This suggests that, to improve on blood circulation, one need to constantly subject the body to moderate magnetic field intensity.

Blood does not exhibit a constant viscosity at all flow rates and can be regarded as non-Newtonian and it is best studied under the field of bio-rheology. The blood flow velocity effectively describes everything about the motion of blood in the circulatory system with many of the properties expressed mathematically in terms of the flow velocity. At the same time for a non-Newtonian fluid, the viscosity is determined by the flow characteristics. Figures 14, 15 and 16, shows the variation of velocity profile of the three-layered fluid model along the radial distance for different values of magnetic field intensity, shear thinning and shear thickening. The velocity field is independent of the z and θ coordinates therefore, it is changing only along the radial distance and the curves shift away from the origin and increase steadily when the shear thinning and magnetic field intensity increase (Figures 14 and 15), and begin to move towards the origin when the shear thickening increases (Figure 16). This shows that the velocity increases with increase in shear thinning and magnetic field and reduce with increase in shear thickening.

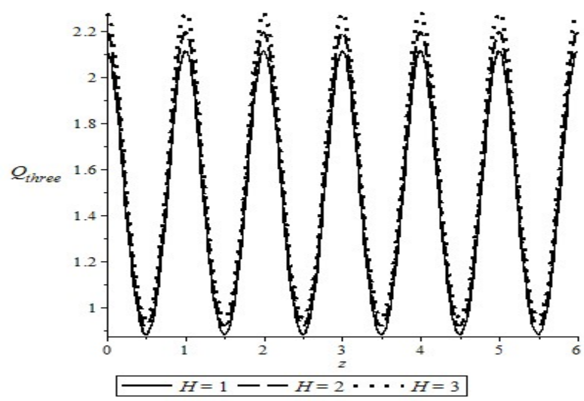

Figure 2: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Magnetic Field Intensity in the Axial Direction after the Stenosis Position.***

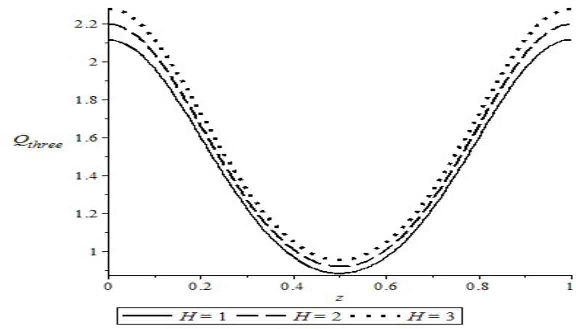

Figure 3: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Magnetic Field Intensity in the Axial Direction before the Stenosis Position.***

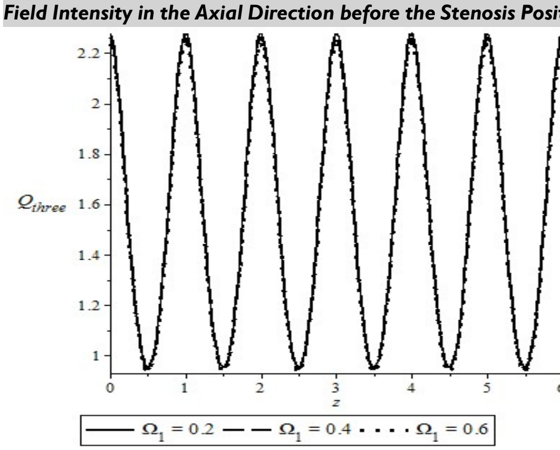

Figure 4: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thickening in the Axial Direction after the Stenosis Position.***

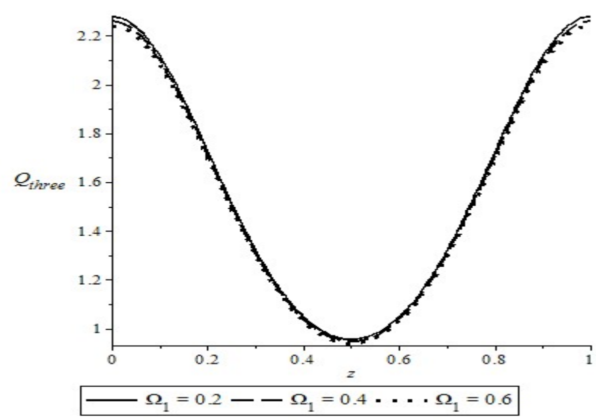

Figure 5: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thickening in the Axial Direction before the Stenosis Position.***

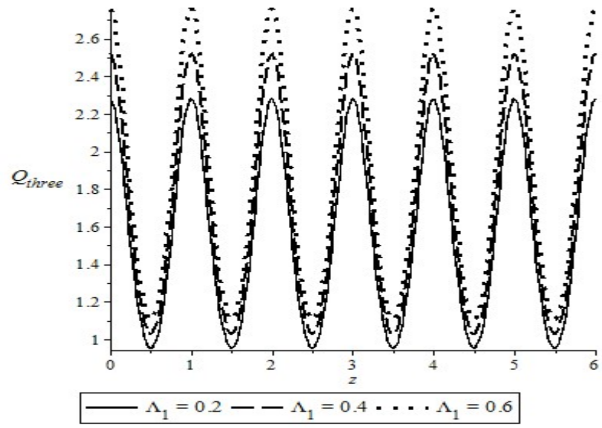

Figure 6: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thinning in the Axial Direction after the Stenosis Position.***

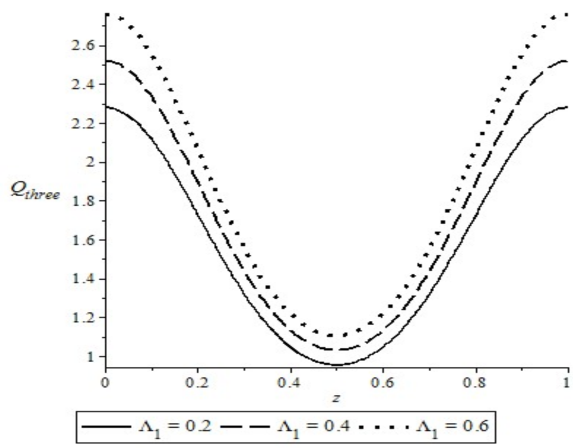

Figure 7: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thinning in the Axial Direction before the Stenosis' Position.***

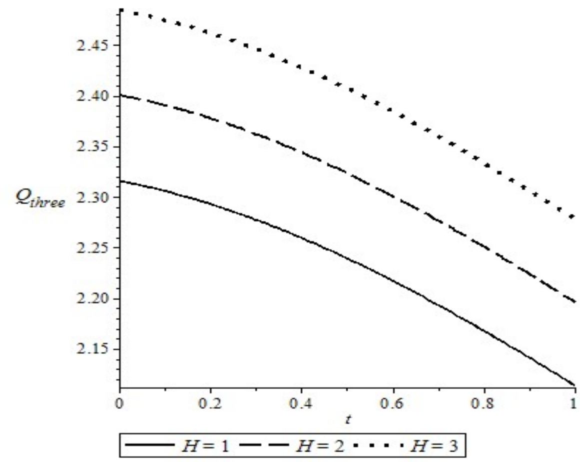

Figure 8: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Magnetic Field Intensity over Time.***

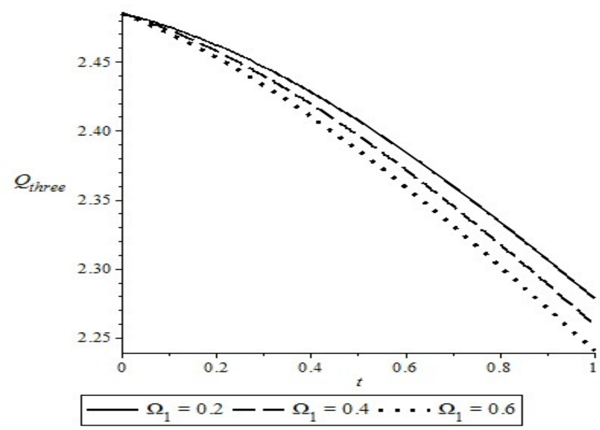

Figure 9: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thickening over Time.***

Blood as a fluid also known as pseudo-plastics, exhibit both shear thickening and shear thinning rheological properties. A fluid is said to be shear thickening if the viscosity of the fluid increases as the shear rate increases and is said to be shear thinning if the viscosity decreases as the shear rate increases. Generally, blood flow is highly dependent on the viscosity which determines the shear thinning and thickening of the blood. Increases in the shear thickening reduce the volume flow rate as seen by Figures4, 5 and 9 and increase the resistance (Figure 12). Whereas, increase in the shear thinning improves the volume flow rate to a great measure (Figures 6, 7 and 10) offering a low resistance to the flow (Figure 13).

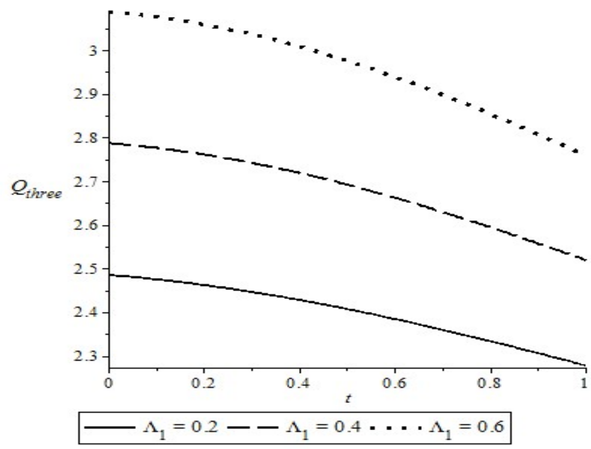

Figure 10: ***Variation of the Volumetric Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thinning over Time.***

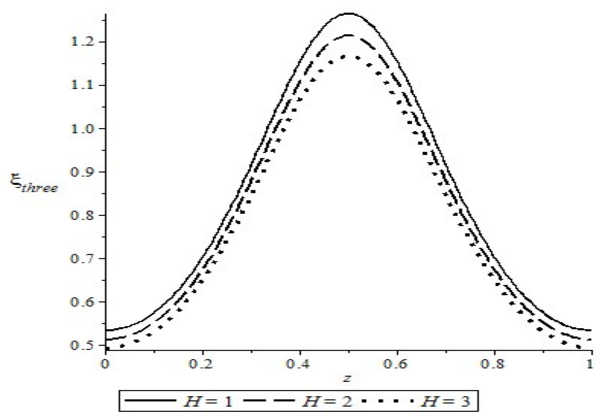

Figure 11: ***Variation of the Resistance to the Total Volume Flow Rate of the Three-Layered Blood Flow Model with Different Values of Magnetic Field Intensity in the Axial Direction before the Stenosis' Position.***

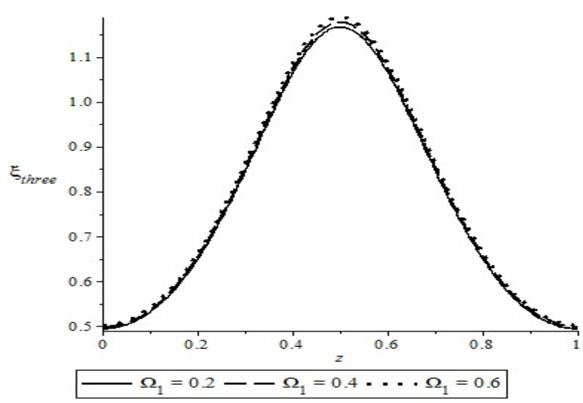

Figure 12: ***Variation of the Resistance to the Total Volume Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thickening in the Axial Direction before the Stenosis' Position.***

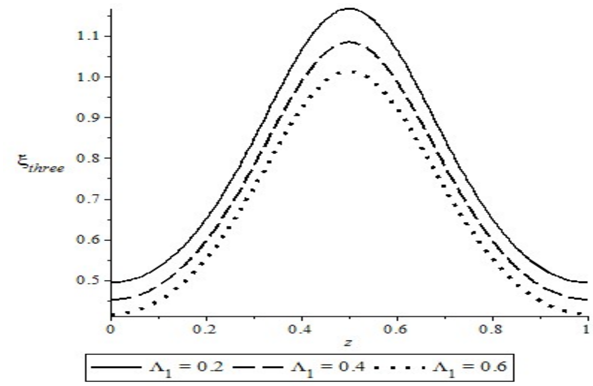

Figure 13: ***Variation of the Resistance to the Total Volume Flow Rate of the Three-Layered Blood Flow Model with Different Values of Shear Thinning in the Axial Direction before the Stenosis Position.***

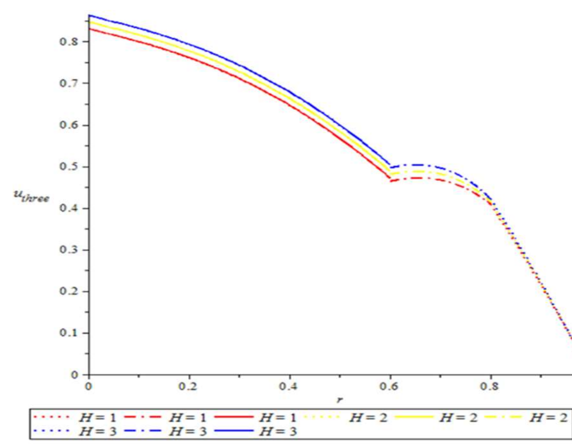

Figure 14: ***The Velocity Profile of the Three-Layered Blood Flow Model with Increasing Magnetic Field Intensity.***

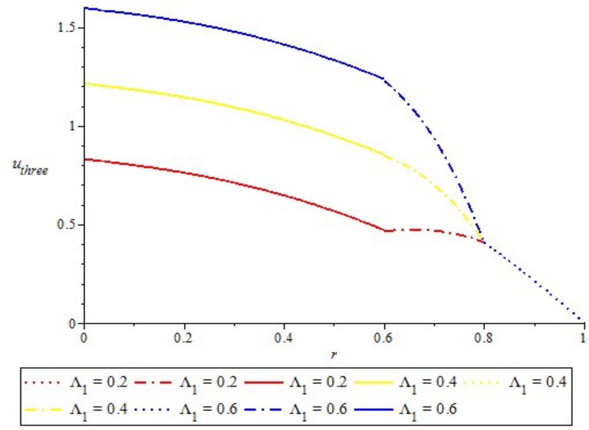

Figure 15: ***The Velocity Profile of the Three-Layered Blood Flow Model with Increasing Shear Thinning***

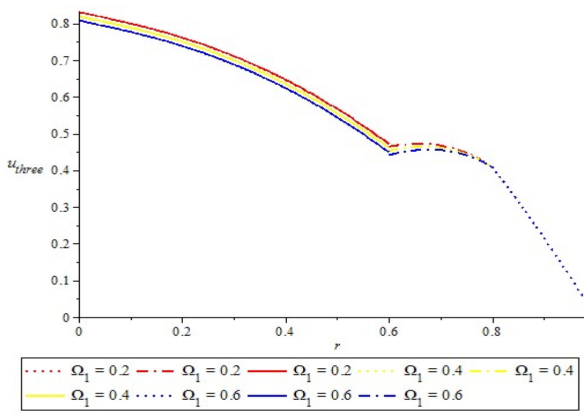

Figure 16: ***The Velocity Profile of the Three-Layered Blood Flow Model with Increasing Shear Thickening.***

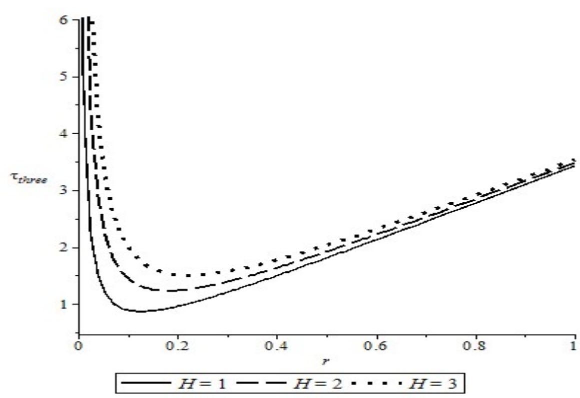

Figure 17: ***The Shear Stress of the Three-Layered Blood Flow with Increasing Magnetic Field Intensity in the Radial Direction.***

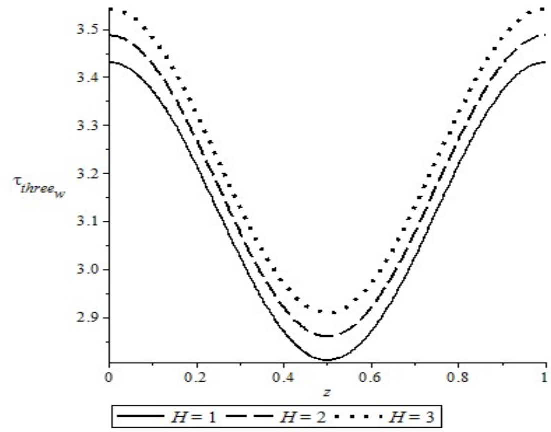

Figure 18: ***The Wall Shear Stress of the Three-Layered Blood Flow with Increasing Magnetic Field Intensity in the Axial Direction.***

The shear stress at the wall that is associated with blood flow through an artery depends on the artery size and geometry (Potters, 2014). By this model, the shear stress is not determined by the shear thinning or shear thickening. However, the shear stress increases with increase in magnetic field intensity as seen by Figures 17 and 18.

Under normal conditions, shear stress maintains its magnitude and direction within an acceptable range. While the direction of the stress may also change by the reverse flow, depending on the hemodynamic conditions. This supposes that the shear stress is in the direction of flow as can be seen by Figure 18.

Conclusion

This research reveals that one does not need to subject the whole body under the influence of a magnetic field to improve the blood volume flow rate. In fact, subjecting an artery in the magnetic field along the direction of flow for a few minutes increases the volume flow rate. Furthermore, increasing the magnetic field intensity continuously increases the volume flow rate as can be suggested in Figures 8 and 9 which increase the velocity of blood and the circulation would transmit the effect to whole body. This will constantly check the pressure force exerted by the heart, reducing its risk of total failure.

This remains an interesting research. However, it is based on theoretical findings. Experimental results will further validate this concept for clinical benefits.

Declaration of conflicting interests

The author declared no potential conflicts of interest.

References

[1] Buchanan Jr., J. R., Kleinstreuer, C. and Corner, J. K. (2000). Rheological Effects on Pulsatile Hemodynamics in a Stenosed Tube. *Journal of Computers and Fluids*, 29: 695-724.

[2] Chaturani, P. and Biswas, D. (1983). Three- Layered Couette Flow of Polar Fluid with Non-Zero Particle Spin Boundary Condition at the Interfaces with Applications to Blood Flow. *Journal of Biorheology*, 20(6):733- 44.

[3] Dharmendra, T. (2012). A Mathematical Study on Three Layered Oscillatory Blood Flow through Stenosed Arteries. *Journal of Bionic Engineering,* 9: 119–131.

[4] Fernando, C. (2008). Axisymmetric Motion of a Generalized Rivlin-Ericksen Fluids with Shear-dependent Normal Stress Coefficients. *International Journal of Mathematical Models and Methods in Applied Sciences*, 2(2): 168- 175.

[5] Hayat, T., Anum, S. and Alsaedi, A. (2015), MHD Axisymetric Flow of Third Grade Fluid by a Stretching Cylinder. *Alexandria Engineering Journal*, 54: 205-212.

[6] Ku, N. D., (1997), Blood Flow in Arteries. *Annual Review Fluid Mechanics*. 29: 399–434

[7] Pandey, S. K., Chaube, M. K. and Dharmendra, T. (2011). Peristaltic Transport of Multi-Layered Power-Law Fluids with Distinct Viscosities: A mathematical Model for Intestinal Flows. *Journal of Theoretical Biology*, 278: 11–19.

[8] Pijush, K. K. and Ira, M. C. (2008). Fluid Mechanics. Fourth Edition. Academic Press, Sydney-Tokyo.An Imprint of Elsevier. P. 782.

[9] Rekha, B. and Usha, A., (2015). A Casson Fluid Model for Multiple Stenosed Artery in the Presence of Magnetic Field, *E-Journal of Science and Technology* (e-JST).10(4), 53-64

[10] Sandoo, A. V., Zanten, J. J., Metsios, G. S., Carroll. D. and Kitas, G. D. (2010) The Endothelium and its Role in Regulating Vascular Tone. *Open Cardiovascular Medical Journal.* (302) 12.

[11] Sankar, D. S. (2009). A Two Fluid Model for Pulsatile Flow in Catheterized Blood Vessels. *International Journal of Non-Linear Mechanics* 44: 37-351.

[12] Sankar, D. S. and Hemalatha, K., (2006). Pulsatile Flow of Herschel–Bulkley Fluid through stenosed arteries—A mathematical model Article. *International Journal of Non-Linear Mechanics* 41(8):979-990

[13] Sankar, D. S. and Lee, U. (2009). Mathematical Modeling of Pulsatile Flow of Non-Newtonian Fluid in Stenosed Arteries. *Communications in Nonlinear Science and Numerical Simulation,* 14(7): 2971-2981.

[14] Tortora, G. J. and Derrickson, B. (2012). *The Cardiovascular System: Blood Vessels and Hemodynamics. Principles of Anatomy and Physiology*. 13th Edition. John Wiley and Sons, Hoboken, Pp. 729–732.

[15] Tzirtzilakis, E. E. (2005). A Mathematical Model for Blood Flow in Magnetic Field. *Physics of Fluids*, 17(077103): 1-15.

[16] Zeb, M., Islam, S., Siddiqui, A. M. and Haroon, T. (2013). Analysis of Third-Grade Fluid in HelicalScrew Rheometer. *Journal of Applied Mathematics*, 1- 11.

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