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Stochastic Modelling of Oil Spill Incidences as Renewal Process

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Abstract

Oil spills constitute a major source of threat to livelihood in the Niger Delta, Nigeria. It exerts deleterious impact on the entire ecosystem. This paper presents a stochastic model based on renewal theory for modelling oil spill incidences. It considered oil spill incidences as a counting process whose inter-event times are independent and identically distributed exponential variables. The model was implemented on oil spills data from CHEVRON and TOTAL operations that was recorded from January, 2006 to May, 2014. The study revealed that the mean time to oil spills from CHEVRON Nigeria Limited and TOTAL Nigeria Limited operations were 6 and 43 days respectively**.** Estimates of occurrence of oil spills are helpful preliminary information for analysing impact of oil spills.

Keywords: Oil Spills, Renewal Theory, Poisson Process, Stochastic Process, Niger Delta.

Introduction

According to the Central Bank of Nigeria's Balance of Payment (BOP) brief, oil and gas sector accounted for 93.8% of the country's total export revenue in the 1st quarter of 2018. Crude oil accounted for 85% of Nigeria's gross domestic product (GDP) and over 80% of the country's revenue [1]. The Niger Delta region, which possesses Africa's biggest crude oil reserve is the main crude oil production region in Nigeria [2]. Nigeria began commercial exploration of crude oil in 1958 when the Royal Dutch Shell started exportation of oil from Oloibiri and Afam oil fields in Port Harcourt, Rivers State of Nigeria [3].

Crude oil exploration in the Niger Delta is characterized by incessant oil spills. [4] reported that about 7,000 oil spills occurred in the Niger delta between 1970-2000. In another study, it was revealed that a total of 6,288 oil spill incidences which amounted to 211, 377.17 barrels were recorded between January, 2006 to May, 2014 [5]. An independent assessment conducted by United Nations Environment Programme (UNEP) on the environmental and public health impacts of crude oil contamination in Ogoniland presented 67 contaminated oil spill sites [6]. These are just few statistics on oil spill incidences besides several unreported cases. The release of liquid petroleum hydrocarbon into the environment as a result of human activity is known as oil spill.

Many researchers have pointed to the direct and indirect consequences of crude oil spills in the Niger Delta Region [7-11]. Many, whose livelihood depend on farming have been rendered unemployed. One of the remote resultant effects of these 'idle minds' in the Niger Delta is worsening insecurity. Oil spills are sources of setback to Nigeria's attainment of Sustainable Development Goals 2, 3, 6, and 11 which are: zero hunger through sustainable agriculture, good health, clean water and sanitation, and sustainable cities free of insecurity respectively. [12] regrouped the impact of oil spills into socio-economic impacts, physical-health impacts and socio-cultural impacts. Although there are many legislations that could curb the menace of oil spill in Nigeria, poor management practices by oil industries and the neglects of regulatory agencies contribute to the over-bearing of the ecosystem with incessant oil spills.

Based on UNEP's recommendation, the federal government of Nigeria on June 2, 2016, flagged-off a $1 billion clean-up and restoration of Ogoniland in the Niger Delta. Although the Ogoniland clean-up was to last for a period of 5 years, experts opined that it may take about 25 years for the ecosystem to be fully restored. Worthy of note is that the said clean-up is only to be carried out in Ogoniland which covers the area of 1, 000 km^2; about 1.3% of the 75,000 km^2 of the Niger Delta region [13]. Nonetheless, with the UNEP's report on the environmental assessment of Ogoniland, one can imagine the horrendous impact of oil spill in the entire Niger Delta region on Nigeria.

Given the enormity of the negative implications of oil spills in the oil rich Niger Delta, there has been growing research interest to describe its occurrences. For instance, researchers have made attempts to provide information on the incidences, causes, economic and environmental impacts, and security implications of oil spills in the Niger Delta region [14-17]. However, quantitative modelling of oil spill related data appears to be scanty in literature. Few authors have considered the theory of reliability in isolating risk factors that are associated with crude oil spills in the

Niger Delta [5,13]. More recently, [19] used a two-state Markov model to predict the occurrence of oil spills in the Niger Delta Region using crude oil spill data from Shell Nigeria database from 2015-2018. Elsewhere, risk assessment of oil spills has been studied based on conditional probability technique. These models sought to measure the probability of oil spill impact on its immediate environment [20-23].

The aim of this paper is to model oil spill incidences using renewal theory. In this context, renewal process is defined as a counting process which the inter-event times between oil spills are independent and identically distributed. It is assumed that an oil exploration process begins at time $(t) = 0$, then exploration process fails due to oil spill at some random time period of time $X_1 > 0$, and the system is repaired, another round of exploration commences and last for another random period of time X_2 but with the same distribution as X_1 and so on. The fact is that the process of oil exploration starts over, interdependently of the past spills, after every repair of the system. The remaining part of the paper is organised as follows: the renewal theory is described in Section 2; along with an outline of solution procedure based on the oil spill data. Section 3 presents the results obtained by the application of the procedure in Section 2 on oil spill data. In Section 4, the discussion of results is presented and concluding remarks are made in Section 5.

Methods

Renewal theory

In this section, the theory of renewal process is presented due to [24], beginning with key notations and assumptions.

Notations

$N(t)$ = a counting process that records the number of oil spills that occurred by time t.

X_i = the i^{th} inter-event time.

$S_n = \sum_{i=1}^{n} X_i$ = the time until the n^{th} oil spill.

$S_0 = 0$

$F(x)$ = common distribution function of X_i.

$\mu = E(X_n)$ = mean time between successive renewals.

$m(t) = E[N(t)]$ = Expected number of oil spills by time t.

Assumptions

i. $X_i; i = 1,2,\cdots$ are independent and identically distributed.
ii. $X_i; i = 1,2,\cdots$ are exponentially distributed.
iii. The system of oil exploration is renewed after each repair after oil spill.

Definition 1: Let $X_1, X_2, \cdots$ be a sequence of nonnegative random variables that are independent and identically distributed, then the counting process $\{N(t), t \geq 0\}$ is a defined as a *renewal process*.

Definition 2: A counting process with independent and identically distributed exponential inter-event times is known as a *Poisson process*.

Consider the common distribution function of X_i which is given as

$$F(x) = P(X_i \leq x) \quad (1)$$

A counting process can be defined as

$$N(t) = \max\{n: S_n \leq t\} \quad (2)$$

The distribution of $N(t)$

To obtain the distribution of $N(t)$ we first note that $N(t) \geq n$ if and only if the n^{th} arrival occurs by time t; that is

$$N\{(t) \geq n\} = \{S_n \leq t\} \quad (3)$$

Lemma 1: For any $t > 0$ and $n = 0, 1, 2, \cdots$

$$P[N(t) = n] = P(S_n \leq t) - P(S_{n+1} \leq t) \quad (4)$$

Proof

From Equation (3) we have that

$$P[N(t) = n] = P[\{N(t) \geq n\} \cap P[N(t) \geq n+1] \quad (5)$$
$$= P[S_n \leq t] - P[S_{n+1} \leq t] \quad (6)$$

Therefore,

$$P[N(t) = n] = F_n(t) - F_{n+1)}(t) \quad (7)$$

To obtain the distribution of a renewal process with exponentially distributed random variables we state the following lemma without proof. Interested readers are referred to [24] for the proof.

Lemma 2: Suppose $N(t)$ is a renewal process having exponentially distributed inter-event times with parameter $\lambda > 0$, i.e. $P(X_i \leq t) = 1 - e^{-\lambda t}$, Then $N(t)$ is Poisson distributed with mean λt which implies

$$P[N(t) = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \quad ; n = 0,1,2,\cdots \quad (8)$$

The reverse of Lemma 2 is equally true. That is, if a counting process $N(t)$ is Poisson, then the inter-event times is exponentially distributed.

The renewal function $m(t)$: The renewal function represents the expected value of $N(t)$ and it is obtained by conditioning on the first event of the process. Hence,

$$\mathrm{m(t) = E[N(t)] = \int_0^\infty E[N(t)/X_1 = x]\, f(x)dx} \quad (9)$$

Note that

$$E[N(t)/X_1 = x] = \begin{cases} 0 & ;x > t \\ 1 + E[N(t-x)] & ;x \le t \end{cases} \quad (10)$$

$$m(t) = \int_0^t \big(1 - m(t-x)\big) f(x)dx \quad (11)$$

Expanding Equation (11) gives the desired result;

$$m(t) = F(t) + \int_0^t f(x)m(t-x)dx \quad (12)$$

Equation (12) is a renewal equation for continuous inter-event time distributions.

For a counting process $N(t)$ that is Poisson with parameterλt, $F(t) = 1 - e^{-\lambda t}$ and $f(t) = \lambda e^{-\lambda t}$, Equation (12) becomes

$$m(t) = F(t) + \int_0^t \lambda(t-x)f(x)dx \quad (13)$$

$$= F(t) + \int_0^t \lambda(t)f(x)dx - \int_0^t \lambda x f(x)dx \quad (14)$$

$$= (1+\lambda t)F(t) - \lambda^2 \int_0^t x e^{-\lambda x}dx \quad (15)$$

Integrating Equation (15) by parts and also noting that $\int_0^t x e^{-\lambda x}dx = F(t)$ yields

$$= (1+\lambda t)F(t) + \lambda t e^{-\lambda x} - F(t) \quad (16)$$

Therefore;

$$m(t) = \lambda t \quad (17)$$

Limit behaviour of the renewal process

Sometimes one may be interested in knowing the average renewal rate in the long run. Theorems 1 presents the limiting behaviour of the renewal process without proof. The proof of this theorem is based on the popular law of large numbers. Interested readers may refer to [25] for the proof of theorem 1

Theorem 1: Suppose $\{N(t), t \ge 0\}$ is a counting process, with probability 1

$$\frac{N(t)}{t} \to \frac{1}{\mu} \quad \text{as } t \to \infty. \quad (18)$$

Theorem 1 means that with probability 1, the average renewal rate up to time t converges to $\frac{1}{\mu}$ as $t \to \infty$

Data: The data consist of reported cases (with dates) of oil spill incidences from the operations of CHEVRON Nig. Ltd and TOTAL Nig. Ltd from January, 2006 to May, 2014. This implies that the study period (T) is approximately 3068 days. A total of 514 cases of oil spills were reported from the operation of CHEVRON while 66 cases came from TOTAL. The data was sourced from National Oil Spill Detection and Response Agency (NOSDRA); the agency which is under the Ministry of Environment, charged with the responsibility of detecting of crude oil spills and monitoring responses by oil producing companies. Using the available information, inter-event times before the occurrence of all recorded oil spills were generated. The analysis was carried out with the help of EasyFit 5.5 and Microsoft Excel.

Results and Discussion

Recall that $N(t)$ is a counting process and also considering Lemma 2, the data sets were modelled using exponential distribution.

Figure 1 presents the probability density function of the exponential distribution with parameter $\lambda_a = 0.1683$ which was obtained from that data of CHEVRON Nigeria Limited. Figure 2 also presents the probability density function of the exponential distribution with parameter $\lambda_b = 0.0228$, obtained from oil spills data of TOTAL Nigeria Limited.

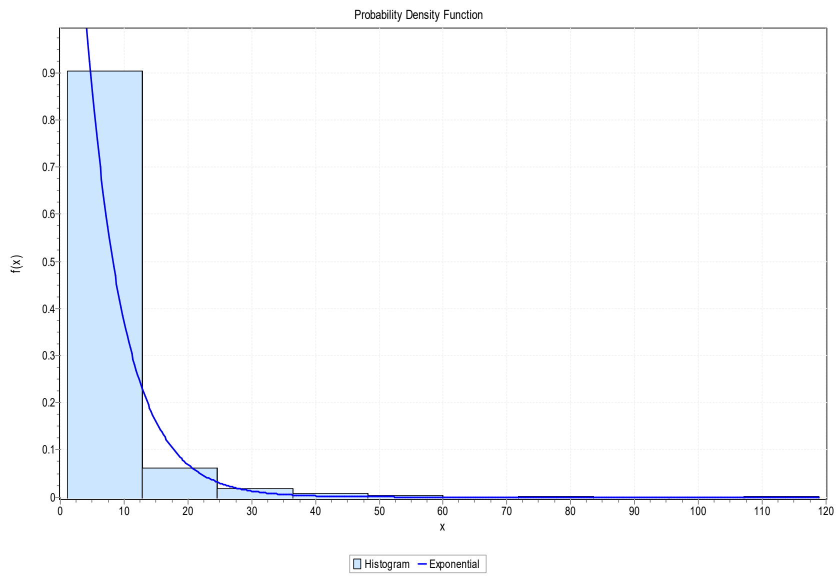

Figure 1: Probabilty Density Function for CHEVRON

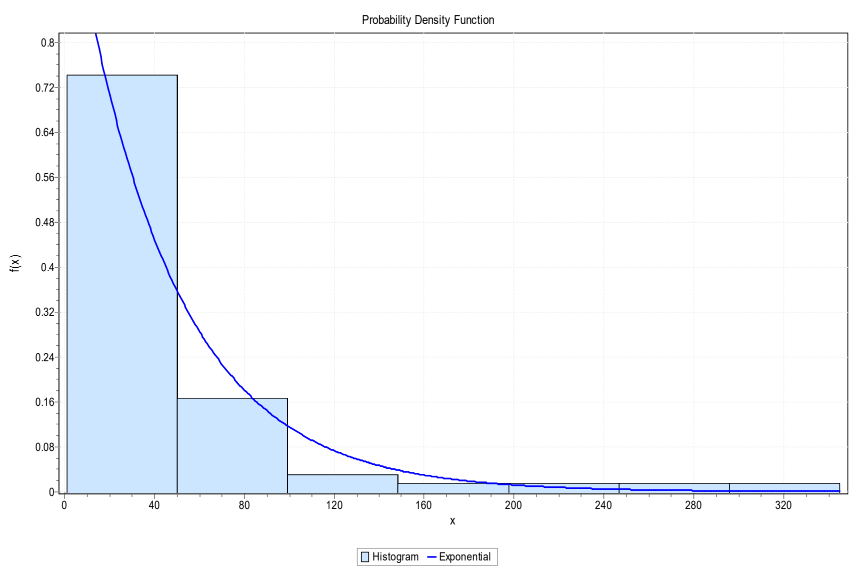

Figure 2: Probabilty Density Function for TOTAL

Given that $\lambda_a = 0.1683$ and noting Equation (17), the renewal function for CHEVRON is

$$\mathrm{m_a(t) = 0.1683t} \;\; ; \mathrm{t} \geq 0 \qquad (20)$$

Since the renewal process is a Poisson process with rate $\lambda_a = 0.1683$ and the period of time (t) under review is approximately 3068 days

$$P\{N_a(3068) = n\} = \frac{e^{-516}(516)^n}{n!} \;\; ; n = 0,1,2,\cdots \qquad (21)$$

where $N_a(t)$ and $m_a(t)$ denote the counting process and renewal function for CHEVRON respectively.

The renewal function for the data from TOTAL is

$$m_b(t) = 0.0228t \;\; ; t \geq 0 \quad (22)$$

and

$$P\{N_b(3068) = n\} = \frac{e^{-70}(70)^n}{n!} \quad ;n = 0,1,2,\cdots \quad (23)$$

where $N_b(t)$ and $m_b(t)$ denote the counting process and renewal function for TOTAL respectively.

Since the means, $\lambda_a t = 516$ and $\lambda_b t = 70$ of the Poisson distributions which represent the expected number of oil spills for CHEVRON and TOTAL are large, the Poisson approximation to Normal distribution is considered. In other words,

if $X \sim Poisson(\lambda t)$ and $\lambda t \rightarrow \infty$, then $X \sim N(\lambda t, \lambda t)$ i.e.

$$Z_{\lambda t} = \frac{N(3068) - \lambda t}{\sqrt{\lambda t}} \sim Z(0,1) \quad (24)$$

Table 1 presents estimate of some probability values and mean renewal times. Probability statements provide further information on the incidence of oil spills. For instance, row 3 of Table 1 states that the probability of recording more than 516 oil spills within the period under study is 0.4920. This implies that the probability of having more than the expected number of oil spills ($m(t)$) is 0.4920. Similarly, row 3 of Table 1, says that the probability of recording more than the expected oil spills ($m(t)$ =70) is 0.4761. The mean time to oil spills (μ) for CHEVRON is 6 days while that of TOTAL is 43 days. Based on Theorem 1, It is possible to assert with probability 1 that at least a spill will occur in CHEVRON operations within 6 days as $t \rightarrow \infty$. For TOTAL operations, at least an oil spill will be recorded within 43 days as $t \rightarrow \infty$.

Table 1: Estimate of Probabilities and Mean Renewal Time

CHEVRON	TOTAL
$P\{N(3068) = 500]$ = 0.0156	$P\{N(3068) = 60]$ = 0.023
$P\{N(3068) > 516]$ = 0.4920	$P\{N(3068) > 70\,]$ = 0.4761
$P\{400 \leq N(3068) \leq 500]$ =0.2643	$P\{60 \leq N(3068) \leq 70]$ = 0.3490
$\mu_a = 6$	$\mu_a = 43$

Conclusion

This work has demonstrated how oil spill incidences could be modelled as renewal process. The model which considered exponentially distributed inter-event times produced reliable results that presented a clear picture of oil spill incidences from CHEVRON and TOTAL operations in the Niger Delta. The study has revealed that the mean time to oil spills form CHEVRON and TOTAL Nigeria operation are 6 and 43 days respectively. Estimates of occurrence of oil spills are helpful preliminary information for analysing impact of oil spills and possible remediation. However, it is recommended that further studies should be directed towards isolating the oil spills based on its magnitude vis-a-vis the operating companies in order to have better understanding of the impact of oil spills.

Declaration of conflicting interests

The author declared no potential conflicts of interest

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